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ABSTRACT

The purpose of this text is to improve the mathematics performance of water treatment operators. The text is divided into two sections. The first section involves fundamental mathematics; the second section involves the process calculations at a plant. (Author/CO)

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MATHEMATICS

... FOR WATER SUPPLY OPERATORS

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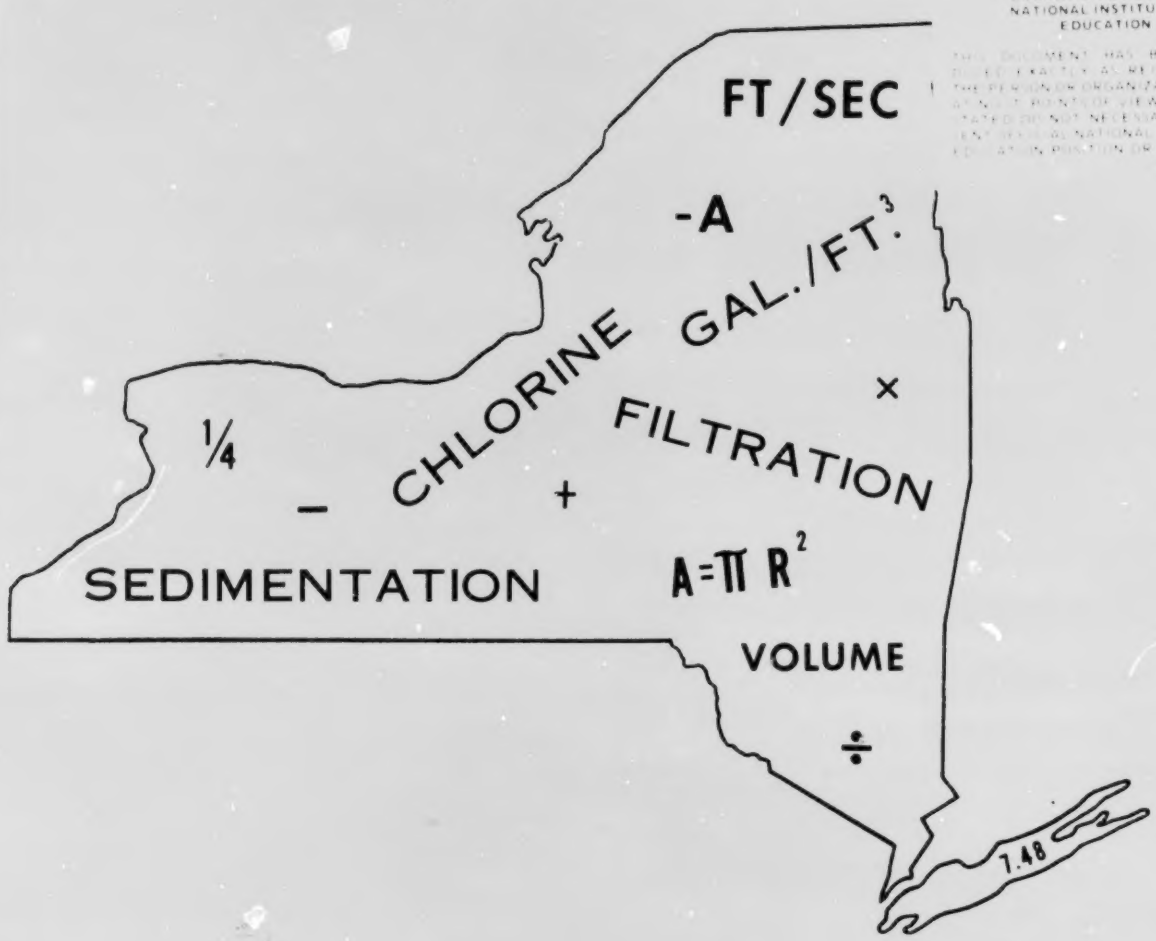

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MATHEMATICS FOR WATER SUPPLY OPERATORS

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INTRODUCTION

An area of concern in the water treatment field is the inability of plant operators to perform the mathematics associated with their job. The purpose of this text is to improve the operator's performance.

The text is divided into two sections; the first dealing with fundamental mathematics, and the second dealing with the process calculations at a plant. Throughout the text example problems and practice problems represent actual water treatment plant operations problems. The text is a practical application of mathematics in a water supply facility.

The text is designed to be used as a correspondence course. That is, the student sets his or her own rate of study. We suggest you read the chapter, study the example problems, do the practice problems, check your answers to the practice problems and then do the chapter assignment. The assignment is then graded, and returned to you with whatever corrections are necessary.

The instructor besides grading the course assignments should be available to you for whatever assistance is necessary. Do not allow a moment of confusion to ruin your entire course of study. Please call or write the instructor for help. The instructor will also visit you at your work site to answer questions and to help you apply what you learned to your actual job responsibilities.

You are cautioned, however, that to attain anything worthwhile usually involves some sacrifice and hard work. So it is with advancing in one's chosen occupation. This course has been designed to help you learn, through diligent and patient study of the material. The instructor will assist you as much as possible, but in the final analysis the work must truly be your own.

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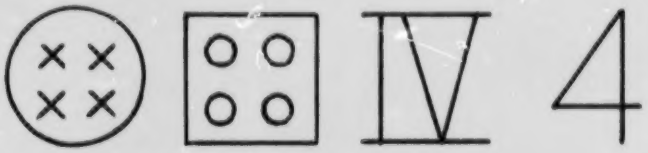
MATHEMATICS FOR WATER SUPPLY OPERATORS

SECTION I CHAPTER I

A. NUMBERS & NUMERALS

> Numbers are an idea or concept.
> Numerals are symbols which we use to represent or express numbers.

The number four (4) can be represented in many ways as shown below:



The symbols or numerals we use in mathematics are called the Arabic system of numerals.

This system has ten (10) different numerals to represent ten (10) different ideas or numbers or quantities.

0 1 2 3 4 5 6 7 8 9

Each progressive number contains one (1) more quantity than the preceding number.

5 is one more than 4

9 is one more than 8

1 is one more than 0

When we want to represent a number that is larger than nine, we use the basic numbers in combination. This gives us numerals of more than nine (9).

10 gives us 1 more than 9

11 gives us 1 more than 10

12 gives us 1 more than 11

13 gives us 1 more than 12

and so on.

When we want to represent a number larger than 99 we use the following combination:

100 gives us 1 more than 99

101 gives us 1 more than 100

etc. . .

The first column of numbers is called the ones column such as:

1
7
6
5
9

} all in the ones column

When we get a number with two places, such as 12, we then call the column to the left of the ones column - the tens column.

tens	ones
1	6
2	4
7	5
3	2

When we exceed 99 we then move to the left of the tens column to the hundreds column.

hundreds	tens	ones
1	4	7
5	2	6
7	3	8
9	1	1

And when we exceed 999 we move left to the thousands column and so on.

This system will make up a picture as below.

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
6,	3	0	2,	4	1	6

So with only ten (10) numerals, we have worked out a system of numbers that could go on indefinitely.

Example

748

How many hundreds in this number?	7
How many tens in this number?	4
How many ones in this number?	8

So to read the number it would be seven hundred and forty-eight.

The numerals we use are also called digits.

Arranging any number of digits in a side by side arrangement gives us numbers of different quantities.

7,395 = seven thousand-three hundred- and ninety-five

The same digits in another arrangement would be a different quantity.

3,975 = three thousand-nine hundred-and seventy-five.

The subject of mathematics is study of the manipulation of these numerals or digits which have been used to represent numbers.

There are only four basic procedures of numerical manipulation.

B. ADDITION OF WHOLE NUMBERS

Addition is the sum or total of any two or more numbers.

Basic Facts:

1. The property of 0 with respect to addition means that anything added to 0 will still have the same total or sum. Sum meaning the total in addition. The following examples show this:

$$32 + 0 = 32$$
$$26 + 0 = 26$$
$$0 + 5 = 5$$

2. The commutative property of addition means that if A and B are any whole numbers then $A + B = B + A$. To explain it further if A is 24 and B is 26, this property tells us that $24 + 26 = 26 + 24$.

Practice Problem 1.1

If A is 10 and B is 15, then $10 + 15 = 15 +$ _____.

3. In addition the arrangement of the addends (numbers to be added) does not affect the sum.

Examples: $12 + 10 + 43 = 10 + 12 + 43$

$9 + 6 + 12 = 12 + 9 + 6$

Let us now restate the basic facts of addition:

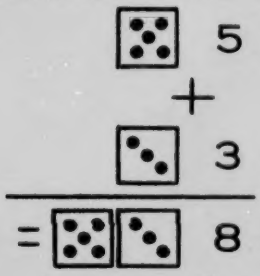
1. Anything added to 0 will still have the same sum or total.

2. If A and B are any whole numbers, then $A + B = B + A$.

3. Order does not affect the sum.

APPLICATION OF BASIC FACTS:

The addition of 5 + 3 can be explained this way:

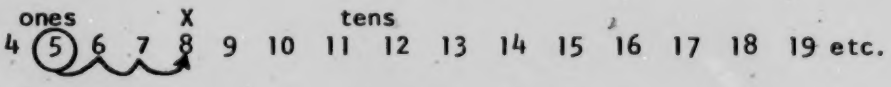


When we add these two boxes (or join them) and count the dots we have eight.

Another way to visualize this concept is to make a chart of our numerals.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 etc.

The above problem 5 + 3 would be solved like this:



Start with the five (5), then move over to the right three (3) digits and you end up with eight (8).

By using this chart, we can solve simple addition problems.

Practice Problems 1.2 - 1.8

Do the following problems with this chart:

2) $\begin{array}{r} 1 \\ +\ 9 \\ \hline \end{array}$ 3) $\begin{array}{r} 7 \\ +\ 8 \\ \hline \end{array}$ 4) $\begin{array}{r} 1 \\ +\ 8 \\ \hline \end{array}$ 5) $\begin{array}{r} 8 \\ +\ 5 \\ \hline \end{array}$ 6) $\begin{array}{r} 2 \\ +\ 6 \\ \hline \end{array}$ 7) $\begin{array}{r} 3 \\ +\ 5 \\ \hline \end{array}$

When the sum becomes larger than the ones column can handle, we move over to the tens column to the left and start to put numbers in that column. Examples: 360, 3650, 36,500, 365,500, 3,650,000.

Using this chart, fill in the other sets of numbers.

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones/Units
				3	6	0
			3	6	5	0
		3	6		0	0

This also happens in the tens column. When the sum becomes larger than the tens column can handle, we move to the left and use the hundreds column.

Practice Problems 1.8 - 1.10

Using the chart add these columns:

8)
1
7
5
+ 3

9)
3
9
1
+ 7

10)
1
6
2
+ 9

This brings up a system of addition that speeds up addition of long columns of figures. It is called adding by tens (10s). Use the system of addition with which you feel the most comfortable.

Example number 1)

1
7
5
+3
16

1 ten
+ 6 left over
16

10
+ 6
16

We pick out the combinations of digits that will add up to ten - put this in the tens column of the answer, then what is left over is added to the total number of tens.

Example number 2)

3
9
1
+ 7
20

2 tens
+ 0 left over
20

20
+ 0
20

Example number 3)

3
9
5
+ 7
24

1 ten
+ 14 left over
24

10
+14
24

Example number 4)

4
6
8
9
2
+ 3
32

2 tens
+ 12 left over
32

20
+12
32

Practice Problem 1.11 - 1.14

Add these columns of digits in this manner - drawing curved lines to show the operation.

11)
6
4
9
1
+ 3

12)
5
4
5
6
+ 3

13)
9
1
3
7
+ 4

14)
8
5
3
7
+ 6

This same system of adding by tens can be used for adding columns of more than one digit. To make it easier for you to follow, use the chart on the right of the page.

341
269
785
775
+ 393
2,563

Thousands	Hundreds	Tens	Units
	3	4	1
	2	6	9
	7	8	5
	7	7	5
	+ 3	9	3
2	5	6	3

You group the digits in the ones column and you find you have twenty-three. Place the 3 in the ones column. Then you take the two tens from there and put it at the top of the tens column. Next group the tens column finding thirty-six. Place the 6 in the tens column and place the three hundreds at the top of the hundreds column. Group the hundreds giving you a sum of twenty-five. Place the five in the hundreds column and the 2 in the thousands column. The sum of the two numbers is 2 thousand 5 hundred and sixty-three.

Practice Problems 1.15 - 1.19.

When doing the problems below you may want to use the chart as shown above.

15)
	6	2
	4	9
+ 5	2	1

16)
	3	2
5	6	4
3	2	4
+	7	1

17)
	2	3
1	5	6
3	2	1
+	7	2

18)
	6	3
1	5	2
	9	5
+ 9	4	6

19)
	9	3
5	0	7
2	1	2
+ 7	1	3

***TIP: When writing whole numbers to be added, always keep the ones in a column, the tens in the next column to the left and hundreds next column to the left, etc. These will always form straight columns.

C. SUBTRACTION OF WHOLE NUMBERS

Subtraction is the taking away or removing of a certain number of objects from a total number of objects to acquire a remainder of objects.

Basic Facts:

1. Property of 0 in subtraction means that for any whole number A, A - 0 = A, since nothing has been taken from A. In the following, 5 - 0 = 5, since nothing is taken from the 5.

Other examples of this property:	15 - 0 = 15 2 - 0 = 2 8 - 0 = 8

2. Remember in our study of addition we had a commutative property, which stated that the order of the addends (numbers to be added) does not affect the sum. Thus we could say A + B = B + A. In subtraction however, there is no commutative property. Suppose we have the subtraction of 8 - 5? We ask the question how many do we have left if we start with 8 and take away 5. The obvious answer is 3. However, if we start with only 5 objects can we take away 8? No. Thus the quantites (8-5 and 5-8) are different, and we have no commutative property.

3. In addition we also have an associative property which says that the way the addends are grouped does not affect the sum. However, in subtraction this rules does not apply because (6-3) - 2 = 3 - 2 = 1. However, 6 - (3-2) = 6 - 1, thus (6-3) is not equal to 6 - (3-2).

Example: (8-4) - 2 = 4 - 2 = 2

Practice Problem 1.20

8 - (4-2) = 8 - 2 = ____

4. The same number property of subtraction, means that for a number minus the same number the result will equal zero. Such is the case in 47 - 47 = 0, or 385 - 385 = 0.

5. You cannot in subtraction take a larger number away from a smaller number. Thus you cannot subtract 5 - 8; because the 5 is smaller than the 8, nor can you subtract 100 - 200 =.

6. The answer in a subtraction problem is called the difference.

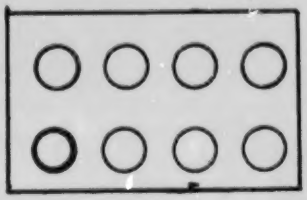
Example: 9 - 4 = 5 which is the difference

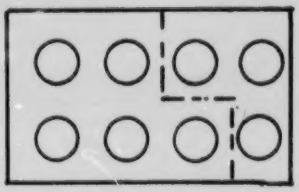
Practice Problem 1.21

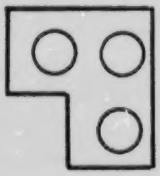
15 - 6 = ____ which is the _____.

APPLICATION OF BASIC FACTS:

Subtraction of 8 - 5 can be explained in this way.

We start with a quantity of 

From this we are going to remove or "take away" a quantity of 

Leaving a quantity of 

So 8 - 5 = 3

Using the same chart of our numerals as we did in addition, we can visualize subtraction (in addition we counted to the right).

In subtraction, since we are reducing or taking away, we count to the left.

8 - 5 = 3

X ones tens

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 etc.

Practice Problem 1.22 - 1.27

Using the numeral chart, do the following subtractions:

22) 9 - 2 23) 7 - 4 24) 14 - 6 25) 12 - 7 26) 16 - 9 27) 11 - 5

The numeral chart we have been using does have some limitations when it comes to larger numbers. Such a chart would simply be too large to be practical. So we return to the column principle of ones, tens & hundreds as we did in addition.

In the problem 36 - 24 =

$$\begin{array}{r} 36 \\ -\ 24 \\ \hline 12 \end{array}$$

we take 4 from 6 in the ones column = 2 then 2 from3 in the tens column = 1 thus 1 ten and 2 ones = 12.

In subtraction there is a problem that comes with the column system. When the number to be reduced in a column is smaller than what we are taking away, we have to borrow a ten or a hundred to use in that column.

$$\begin{array}{r} 3\ \not{4}\,{}^{1}2 \\ -\ 1\ 9 \\ \hline 2\ 3 \end{array}$$

we borrow one ten from 4 tens, leaving three tens and use the one ten with the two ones making twelve and now we can take nine from 12 leaving 3; then we take one ten from 3 tens leaving 2 tens. The answer is 2 tens and 3 ones = 23.

This borrowing has to be done any time the amount to be taken away is more than the amount to be reduced in the column: ones, tens, hundreds, etc.

Working with hundreds, it is done as below:

$$\begin{array}{r} {}^{2}\not{3},{}^{11}\not{2}{}^{16}\not{7}{}^{13}\not{3} \\ -\quad 3\,8\,4 \\ \hline 2,\,8\,8\,9 \text{ ans.} \end{array}$$

When you borrow always cross the number out and write in above a number one less than what you crossed out. To check your answer follow the procedure below:

Checking Answers in Subtraction

In checking a subtraction problem to see if the answer is correct a very simple method exists to do this. In the problem above we subtracted 3,273 - 384.

3,273
- 384
2,889 ans.

To check, take your answer 2,889 and add it to the smaller of the 2 numbers being subtracted. In this case 384.

Example: 2,889 ans.
+ 384 smaller of 2 numbers being subtracted
3,273
Thus we know from the above addition our answer is correct

Example:

3,776
- 470
3,306 ans.

3,306 ans.
+ 470 smaller of 2 numbers subtracted
3,776

Practice Problems 1.28 - 1.33

28) 9 - 3 = ____

29) 12 - 7 = ____

30) 21 - 19 = ____

31) 324 - 49 = ____

32) 2,376 - 1,197 = ____

33) 5,261 - 3,159 = ____

Renaming in Subtraction Problems

In subtraction it may become necessary at times to perform an operation called renaming. By this we mean changing from 1 unit to another. Such as happens in the following examples:

Example:

1 ft. 3 in.
- 9 in.

We cannot perform the subtraction as it is now since we cannot subtract 9 inches from 3 inches. So, therefore, we must change 1 ft. 3 in. to inches.

1 ft. 3 in. = 15 in.
- 9 in.
6 in. ans.

Example:

1 hr. 45 min.	1 hr. 45 min.	= 105 min.
- 50 min.		- 50 min.
		55 min. ans.

Practice Problems 1.34 - 1.36

Do the subtraction in the following examples:

34) 6 ft. 3 in.
- 11 in.

35) 5 ft. 6 in.
-2 ft. 9 in.

36) 18 ft. 2 in.
-8 ft. 8 in.

D. MULTIPLICATION OF WHOLE NUMBERS

Multiplication is the finding of the total number of objects in x numbe: of sets with y number of objects per set.

Basic Facts:

1. Multiplication as in addition has a "commutative property of multiplication". If A and B are any two whole numbers, then A x B = B x A. For the two numbers 9 and 8, this property states 9 x 8 = 8 x 9 or 72 = 72.

2. The two numbers being multiplied are known as factors, and the answer to the multiplication is known as the product.

Example: 7 x 8 = 56 - the factors are 7 and 8 and 56 is the product.

Practice Problem 1.37

4 x 3 = 12 the factors are ___and___. The product is _____.

3. In multiplication we have a property of 1 which means if A is any whole number then A x 1 = 1 x A or 9 x 1 = 1 x 9 or 12 x 1 = 1 x 12.

4. The property of 0 in multiplication means that if A is any whole number then A x 0 = 0 x A, or as in the following example 9 x 0 = 0. Remember that anytime 0 is a factor the product will always be 0.

5. The associative property of multiplication means that if A, B, and C are any whole numbers then (A x B) x C = (B x C) x A or as in the following problem we will know that from the associative property of multiplication that (8x4) x 7 = 8 x (4x7) = 224 or (6x5) x 3 = (3x5) x 6 = 90.

APPLICATION OF BASIC FACTS:

The problem 3 x 2 = can be defined as the total number of objects in three (3) sets with two (2) objects in each set.

3 sets with 2 per set

$$3 \times 2 = 6$$

Another concept of 3 x 2 = 6 is that you add 2 + 2 + 2 = 6

so in turn 4 x 2 = 4 + 4 = 8

or 2 + 2 + 2 + 2 = 8

Practice Problems 1.38 - 1.40

Multiply the following using the addition principle.

38) 4 x 3 =

39) 5 x 3 =

40) 4 x 6 =

A simpler way to multiply is to use multiplication tables which you can now develop since you know the addition principle.

0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

As you notice in the table, 3 x 4 = 12 and 4 x 3 = 12 so it doesn't matter which is multiplied by which, it always gives the same answer. (See Diagram Below)

0	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20

0	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16
5	5	10	15	20

You will notice that for any number multiplied by ten, you add a 0.

In the same way to multiply 100 you add two 00s.

Practice Problems 1.41 - 1.46

Multiply these, and label the factors and products.

41) 27 x 10 = 42) 9 x 10 = 43) 14 x 100 = 44) 4 x 100 = 45) 100 x 9 =

46) 10 x 4 =

We can also use the following method for multiplication by 10's or 100's.

Multiplication of any number multiplied by ten or by 100. In multiplication of this type, all that we have to do is bring the zeroes out as shown below.

Examples

$$\begin{array}{r} 14 \\ \times\ 30 \\ \hline 420 \end{array}$$

the zero is brought out, and then dropped to our answer.

$$\begin{array}{r} 17 \\ \times\ 200 \\ \hline 3400 \end{array}$$

$$\begin{array}{r} 60 \\ \times\ 60 \\ \hline 3,600 \end{array}$$

Multiplication by the Old Method:

Multiplying by a single digit number

Such as: $\begin{array}{r} 39 \\ \times\ 8 \\ \hline \end{array}$ 1- You use the column form to show 39 x 8.
2- You then multiply 9x 8 which equals 72.

$\begin{array}{r} 39 \\ \times\ 8 \\ \hline 2 \end{array}$ You place the two in the ones column and carry the seven over to the tens column to be added to the answer of 8 x 3 which is 24.

$\begin{array}{r} 39 \\ \times\ 8 \\ \hline 312 \end{array}$ 24 + 7 = 31. The one is placed in the tens column and the three in the hundreths column. The resulting answer is three hundred and twelve. (132)

Multiplying a number by another number which has more than one digit.

Such as:

```
  12
x 24
----
  48
+ 24
----
 288
```

1- You proceed exactly as for a single digit multiplier multiply 12 x 4 = 48. Place the eight in the ones column & place the four in the tens column.
2- Then multiply 12 x 2 = 24. You place the four in the tens column and the two in the hundredths column.
3- You then add to get the answer two hundred and eighty-eight (288).

Multipilication by the New Method:

There are two different approaches which we can use in this method. The first is the column system of multiplying, the second grouping the numbers to be multiplied under each column, the ones under the ones, tens under tens and so on. The following examples will illustrate both methods.

Column Method:

```
  39
x  8
----
  72
+240
----
 312
```

1- Multiply 8 ones x 9 ones = seven tens + two ones = 72
2- Multiply 8 ones x 3 tens = 2 hundreds + 4 tens = 240
3- Add the 2 giving 3 hundreds, 1 ten, 2 ones. = 312

Grouping in ones, tens, etc. Method:

Hundreds	Tens	Ones
	3	9
	x	8
	24 tens	72 ones

We now have 24 tens and 72 ones.
We know that 24 tens is 24 x 10 = 240
We now add the 72 ones to it = + 72
312

Use whichever of the 2 methods above that you feel most comfortable with.

Multiplying by a number with more than one digit.

Such as:

```
  12
x 24
----
   8
  40
  40
+200
----
 288
```

4 ones x 2 ones = 8 ones = 8
4 ones x 1 ten = 4 tens = 40
2 tens x 2 ones = 4 tens = 40
2 tens x 1 ten = 2 hundreds = 200
2 hundreds, 8 tens, 8 ones = 288

In this you must remember that after multiplying 4 x 2, then multiply 4 x 10 giving 40. Then you multiply 2 tens or 20 x 2 ones giving 40.

Then you multiply 2 tens or 20 x 1 ten or 20 x 10 = 200.

Placing everything in their proper places of ones, tens, and hundreds makes the addition at the end easier.

Multiplication with 2 three-digit numbers

In multiplying 2 three-digit numbers we find the three partial products and add them together to get our final product. The following illustrations will explain this in more detail.

Example: 374 x 265

1- To get our first partial product we multiply 374 x 5
2- To get our second partial product we multiply 374 x 60
3- To get our third partial product we mutliply 374 x 200

1st Partial Product

$$\begin{array}{r} 374 \\ \times\ 5 \\ \hline 1870 \end{array}$$

2nd Partial Product

$$\begin{array}{r} 374 \\ \times\ 60 \\ \hline 22,440 \end{array}$$

3rd Partial Product

$$\begin{array}{r} 374 \\ \times\ 200 \\ \hline 74,800 \end{array}$$

$$\begin{array}{rl} & 1870 \text{ 1st partial product} \\ & 22440 \text{ 2nd partial product} \\ + & 74800 \text{ 3rd partial product} \\ \hline & 99110 \text{ Product} \end{array}$$

E. DIVISION OF WHOLE NUMBERS

Division means that we are given "x" number of objects and divide or split that "x" number into so many objects per set or group.

Basic Facts

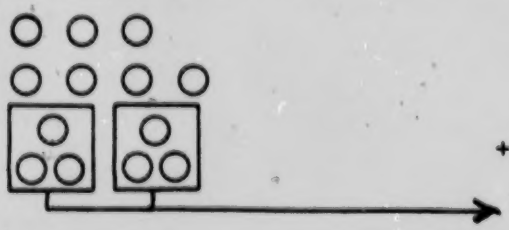
1. The property of 1 with respect to division means that if "N" is any whole number then $N \div 1 = N$ or $5 \div 1 = 5$, or $7 \div 1 = 7$.

2. The property of 0 with respect to division means that if N is any whole number (except 0) then $N \div 0 = 0$. Since we have already seen that $N \div 0 = 0$, we say that division by 0 is undefined, that is we can never divide by 0.

APPLICATION OF BASIC FACTS:

In the concept of dividing 7 by 3, we write either $3\overline{)7}$, $7 \div 3$, $\frac{7}{3}$ to mean 7 divided by 3.

What we are asking is, if we had seven balls and we wanted to put three balls in each box, how many boxes of balls would we have?



The same if we have $15 \div 3$ or $3\overline{)15}$

none left over

5 with no remainder

A division problem is set up this way $2\overline{)\,\overset{5}{10}}$

The 2 is called the <u>divisor</u>, the 10 is called the <u>dividend</u>, and the 5 is called the <u>quotient</u>.

The solving of division problems really ends up being a multiplication problem in reverse.

$5\overline{)30}$ answer 6 $5 \times 6 = 30$

The multiplication table can be used in reverse for division.

For example 30 divided by 5 equals 6:

0	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Example: $3\overline{)\,\overset{150\ R2}{452}}$

-3

15

-15

02

We first divide the 3 into the four which will go 1 time, and place the 1 above the 4. We place the 3 under the 4 and subtract and place a 1 below. Bring down the 5 to make 15. Then we divide the 3 into the 15, getting 5 placing that after the 1 in the answer and subtracting 15 from 15 leaving 0. Bring down the 2. 3 will not go into 2, so we place a 0 after the 5 in the answer and have two left over (Remainder of 2). This is the old system of dividing.

Checking an answer in Division:

In checking an answer in division, simply multiply the divisor by the quotient.

Example:

```
      137  R33
57/ 7842
    57
    214
    171
     432
     399
      33
```

Check:

```
  137
x  57
  959
 685
7809
+  33   remainder
7,842
```

Quotient Divisor

Remember in division problems when checking to add the remainder to the product.

Practice Problems 1.47 - 1.50

Do these division problems and check your answers.

47) 8/ 125 48) 7/ 8,530 49) 3/ 3,005 50) 9/ 7,695

***TIP: Remember to place the number above the portion being divided at the time and work from left to right from thousand to ones.

Working with larger numbers is done the same way as smaller numbers. We can also use the new concept of thousands, hundreds, tens & ones carrying along the zeros to show this.

```
      152 R1
26/ 3,953
    2,600
    1,353
    1,300
       53
       52
        1
```

The only difference to remember is that you are dividing 26 ones into 39. hundreds in the first step.

Second step is 26 ones into 135 tens.

Third step is 26 ones into 53 ones.

Practice Problems 1.51 - 1.54

51) $19\overline{)456}$ 52) $32\overline{)7,956}$ 53) $27\overline{)673}$ 54) $21\overline{)5,384}$

Sometimes in working with larger numbers we may want to use a trial division. Possibly the most difficult part of solving a division problem is in working out a trial division. The following may be helpful to you.

1) In problems like $23\overline{)576}$, $51\overline{)1175}$, etc., where the second digit is 5 or less, use the first digit of the divisor as the guide. In the above, 2 divides into 5 twice, and 5 divides into 11 twice.

2) In other problems such as $29\overline{)667}$ and $37\overline{)1554}$, etc., where the second digit of the divisor is 6 or more, increase the first digit of the divisor by one. In the above 3 divides into 6 twice, and 4 divides into 15 three times.

We do the same thing when the divisor contains more than 2 digits.

Examples of the above two rules are:

```
      24 R19
36/ 763
    62
   143
   124
    19
```

Since 1 is less than 5 we use the 3 as our trial divisor - 3 divides into 7 twice.

```
      143 R24
47/ 6745
    47
   204
   188
    165
    141
     24
```

Since 7 is larger than 5 we use the first digit of the divisor increased by 1 as our trial divisor - 5 divides into 6 once.

Date: _____

Students Name: _____

Chapter 1 Assignment
(To be mailed to Instructor)

Addition:

Add, showing groups of tens:

1)
3
6
7
+9

2)
4
9
3
+7

3)
4
5
8
+1

4)
6
4
4
+6

Add

5)
231
650
420
90
+71

6)
830
713
909
+777

7)
2,311
5,479
2,361
+4,009

8)
3,687,411
231,843
9,278,455
+ 36,119

9) Copy in columns if necessary and add:

23 + 17 + 9 + 15 + 80 = _____

Subtract the following:

10)
1,749
- 958

11)
946
-703

12)
4,561
-2,360

13)
999
-876

14)
27,436
-13,942

(OVER)

In examples 15-23 subtract and check:

15) 3,587,386
-1,310,012

16) 48 - 36 = _____

17) 4,772
-3,772

18) 15,987
-4,986

19) 319
-207

20) 16 hours
-9 hours 10 min.

21) 4 hours 20 min.
-2 hours 30 min.

22) 17 feet 4 inches
-9 feet 4 inches

23) 17 feet 6 inches
-6 feet 9 inches

24 - 26 Multiply using the column method:

24) 798
x53

25) 369
x 8

26) 253
x 30

27 - 29 Multiply using the group method:

27) 253
x300

28) 342
x 6

29) 47
x47

Multiply the following:

30) 260
x528

31) 326
x451

32) 9,137
x 306

33) 7,845
x 176

STUDENTS NAME: _____ DATE: _____

34) 217
×900

35) 398
×527

36 - 40 Divide and check:

36) 70/ 856

37) 50/ 4350

38) 60/ 6239

39) 400/ 8971

40) 42/ 863

Divide:

41) 156/ 1578

42) 15/ 3,000

43) 102/ 73,112

44) 26/ 696

45) Check the following and write "correct" or "incorrect"
Please show all work

A) 3,941 ÷ 33 = 119 R14

B) 3,050 ÷ 231 = 13 R37

SECTION I CHAPTER II

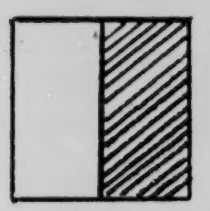
FRACTIONS AND FRACTIONAL NUMBERS

When we use numerals in a certain way to express a portion of one (1), we call the numerals - fractional numbers.

The 1 & 2 used in $\frac{1}{2}$ or 1/2 are fractional numbers

- the top number represents how many equal parts we mean.

$\frac{1}{2}$ - the lower number always represents how many equal parts the whole thing has.



In this square, the whole thing has two equal parts.

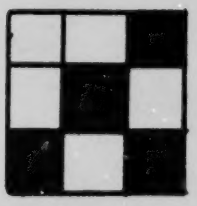
The shaded part is 1 part.

The whole thing has 2 parts so the shaded area is represented by $\frac{1}{2}$ of the whole.

This circle is divided into 3 equal parts.

The shaded part is 1 of 3 equal parts or the shaded area is $\frac{1}{3}$ of the whole.

The unshaded area is $\frac{2}{3}$ of the whole.



This whole square is divided into 9 equal parts.

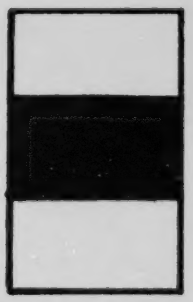
We have shaded 4 of the equal parts.

The fraction we shaded is $\frac{4}{9}$ of the whole.

Practice Problems 2.1 - 2.3

What are the fractional numbers representative for the following shaded parts of the figures.

1)

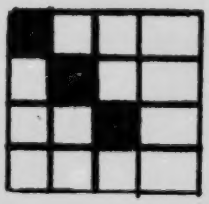


Ans.

2)

Ans.

3)



Ans.

So far, we have used numbers on top of each other with a line drawn between them.

This line is called a fraction bar.

$\frac{\text{numerator}}{\text{denominator}}$ $\quad \frac{1}{6}$ $\quad \longleftarrow$ Fraction Bar

In mathematics, this bar - or slash / separating two quantities always means the top quantity is divided by the lower quantity.

$\frac{3}{4}$ or $3/4$ really means $3 \div 4$ or $4\overline{)3}$

If the numerator and denominator of a fraction are the same, the result is one (1).

$\frac{4}{4} = 1$ $\qquad \frac{8}{8} = 1$

With this same fractional concept, we can represent a whole number in fractions.

$8 = \frac{8}{1}$ $\qquad 4 = \frac{4}{1}$

Units of measure may be expressed as fractions.

For example:

Miles per hour	= mi/hr.
Feet per second	= ft/sec
Million gallons per day	= MG/D or MGD
Gallons per square foot	= gal/ft^2
Milligrams per Liter	= Mg/L

Both numerator and denominator in a fraction may be multiplied by the same number without changing the value of the fraction.

$\frac{3}{5} \times 2 = \frac{6}{10}$ $\frac{1}{3} \times 5 = \frac{5}{15}$ $\frac{3}{4} \times 5 = \frac{15}{20}$

same same same

You may also divide both the numerator and denominator by the same number and not change the value of the fraction.

$\frac{9}{18} \div 9 = \frac{1}{2}$ $\frac{9}{12} \div 3 = \frac{3}{4}$ $\frac{10}{25} \div 5 = \frac{2}{5}$

same same same

Dividing the numerator and the denominator by the same number is called "reducing to simplest terms".

When the numerator is greater than a denominator (such as $\frac{9}{4}$) this is called an improper fraction. The following are improper fractions:

$\frac{3}{2}$ $\frac{4}{3}$ $\frac{5}{4}$ $\frac{10}{3}$ $\frac{21}{6}$

An improper fraction may express a whole number plus a fraction. This is called a mixed number.

Fraction $\frac{9}{4}$ $9 \div 4 = 2\frac{1}{4}$

$4\overline{)9}$ $2\frac{1}{4}$ mixed number

$\frac{8}{1}$ remainder over the denominator

$\frac{1}{4}$

Some mixed numbers are:

$2\frac{1}{2}$ $6\frac{7}{8}$ $1\frac{2}{3}$ $16\frac{1}{4}$ $23\frac{5}{6}$

There are times in this process when we get a mixed number and a fraction that can be reduced to simplest terms.

$\frac{15}{6} = 6\overline{)15}$ 2 $+ \frac{3}{6}$ or $2\frac{1}{2}$

12

$\frac{3}{6}$

$\frac{3}{6} \div 3 = \frac{1}{2}$

32

Practice Problems 2.4 - 2.8

4. What whole number does $\frac{6}{6}$ represent? _____

5. What whole number does $\frac{12}{6}$ represent? _____

6. What mixed number does $\frac{26}{5}$ represent? _____

7. What would $\frac{20}{40}$ be when reduced? _____

8. Reduce to simplest terms $\frac{6}{36}$ = _____

A. ADDITION OF FRACTIONS

The following general rules will apply to both addition and subtraction of fractions.

When combining fractions by addition or subtraction:

1) The denominator of the fractions must be identical.

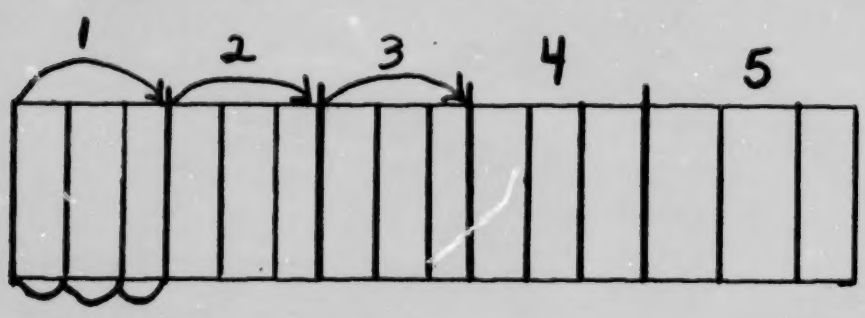
2) The numerators are then combined by addition or subtraction to obtain the new numerator.

3) The new denominator is the same as the like denominators of the original fractions.

Like fractions all have the same number below the fraction bar.

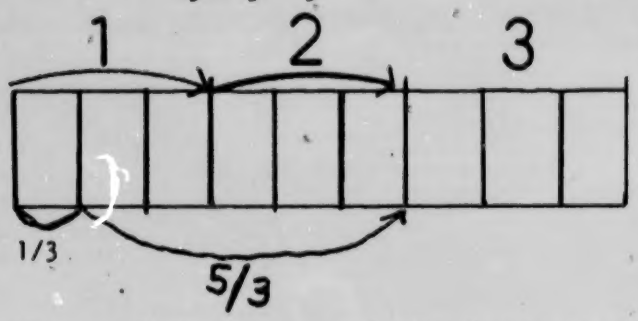
Such as: $\frac{1}{3}$ $\frac{2}{3}$ $\frac{5}{3}$ $\frac{7}{3}$

When we add like fractions we can use the same concept used with whole numbers except each unit is divided into the number below the fraction bar.



$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$ etc.

When we add $\frac{1}{3} + \frac{5}{3} = \frac{6}{3}$ or whole number 2



What we do in the case of like fractions is to add the numerators and place the total over the common denominator.

Such as: $\frac{1}{4} + \frac{3}{4} + \frac{6}{4} = \frac{10}{4} = 4\overline{)\,10}\ 2\ R2 = 2\ 2/4 = 2\ \frac{1}{2}$ (8)

*Don't forget to reduce to mixed number and reduce to lowest terms.

Practice Problems 2.9 - 2.11

Add the following Like Fractions:

9) $\frac{1}{8} + \frac{7}{8} + \frac{3}{8} =$

10) $\frac{3}{16} + \frac{2}{16} + \frac{7}{16} =$

11) $\frac{7}{12} + \frac{3}{12} + \frac{1}{12} =$

Unlike fractions are fractions with different numbers in the denominators.

Some unlike fractions are: $\frac{1}{7}$ $\frac{3}{5}$ $\frac{1}{2}$ $\frac{3}{4}$

Suppose we want to add $\frac{1}{2} + \frac{1}{3}$

Since the denominators are not alike we cannot use the rule for adding like fractions.

But by using a rule about fractions from 3 previous lessons "You may multiply both the numerator and denominator by the same number and not change the value of the fraction."

Such as: $\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$ (same value as $\frac{1}{2}$)

Also: $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$ (same value as $\frac{1}{3}$)

Now both fractions have the same (or a common) denominator.

$$\frac{3}{6} + \frac{2}{6}$$

This we can add with our rule:

$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

This process of converting fractions with different denominators to fractions having the same denominator is called finding the Common Denominator.

The easiest way to find the common denominator is to multiply the two denominators.

$$\frac{1}{3} \times \frac{2}{5}$$

$3 \times 5 = 15$ the Common Denominator

To get the numerators we take the old denominator and divide it into the new common denominator giving you a number which we multiply times the old numerator to give us the new numerator.

$$\frac{1}{3} = \frac{5 \times 1}{15} = \frac{5}{15}$$

And likewise we change:

$$\frac{2}{5} = \frac{3 \times 2}{15} = \frac{6}{15}$$

For example if we want to add $\frac{1}{5} + \frac{3}{4}$

Multiply 5 x 4 = 20 (common denominator)

Divide $5/\overline{20}$ = 4

Divide $4/\overline{20}$ = 5

Thus

$$\frac{1}{5} = \frac{4 \times 1}{20} = \frac{4}{20}$$
$$\frac{3}{4} = \frac{5 \times 3}{20} = \frac{15}{20}$$
$$\frac{19}{20}$$

We usually just write:

$$\begin{array}{r} \frac{4}{20} \\ +\ \frac{15}{20} \\ \hline \frac{19}{20} \end{array}$$

TIP

* When we look for a Common Denominator, we are looking for a number that the Denominators will Divide evenly.

This last tip should always be kept in mind when looking for a Common Denominator.

In the case of adding: $\frac{1}{3} + \frac{5}{12}$

Do not multiply 3 x 12 = 36
Common Denominator

12 is the Common Denominator
Both 3 and 12 divide evenly into 12 -- $3/\overline{12}$ = 4 $12/\overline{12}$ = 1

$$\frac{1}{3} = \frac{4 \times 1}{12} = \frac{4}{12}$$
$$\frac{5}{12} = \frac{5 \times 1}{12} = \frac{5}{12}$$
$$\frac{9}{12}$$

The problem can be worked either way but do not work with figures larger than are necessary.

So to keep with smallest Common Denominators, we take the larger denominator and double it and check if smaller denominators will go into that figure - if not we triple it and try dividing.

Such as: $\frac{1}{6} + \frac{1}{8}$

Will 6 go evenly into 8 - No
Will 6 go evenly into 16 - No
Will 6 go evenly into 24 - Yes - This would be the Common Denominator.

Practice Problems 2.12 - 2.17

What is the common denominator of these pairs of Fractions?

12) $\frac{1}{3}$ and $\frac{3}{4}$ _____

13) $\frac{5}{6}$ and $\frac{3}{4}$ _____

Now add these unlike fractions.

14) $\frac{5}{6} =$

$+\frac{1}{3} =$ _____

15) $\frac{3}{8} =$

$+\frac{1}{3} =$ _____

16) $\frac{1}{9} =$

$+\frac{1}{12} =$ _____

17) $\frac{4}{5} =$

$+\frac{2}{3} =$ _____

We can add any number of unlike fractions as long as we find a common denominator.

Example:
$$\frac{1}{3} = \frac{20}{60}$$
$$\frac{1}{4} = \frac{15}{60}$$
$$+\frac{1}{5} = \frac{12}{60}$$
$$\frac{47}{60}$$

Practice Problems 2.18 - 2.19

Add these groups of unlike fractions.

18)
$$\frac{1}{3}$$
$$\frac{1}{8}$$
$$+\frac{3}{8}$$
_____

19)
$$\frac{1}{6}$$
$$\frac{3}{5}$$
$$+\frac{1}{10}$$
_____

We will now consider adding fractional numbers represented by mixed numerals.

Suppose we want to add $4\frac{1}{7} + 3\frac{3}{7}$.

First change each number to an improper fraction. $4\frac{1}{7} = \frac{29}{7}$ and $3\frac{3}{7} = \frac{24}{7}$

Add the improper fractions:

$$\begin{array}{r} \frac{29}{7} \\ +\ \frac{24}{7} \\ \hline \frac{53}{7} \end{array}$$

Change the answer to a mixed number $7\ \frac{4}{7}$

This is a lot of work for a simple addition problem when we know that $4\ \frac{1}{7}$ really means $4 + \frac{1}{7}$ and $3\ \frac{3}{7}$ means $3 + \frac{3}{7}$

If we add the whole numbers first $4 + 3 = 7$, then add fractions $\frac{1}{7} + \frac{3}{7} = \frac{4}{7}$

The answer is $7 + \frac{4}{7}$ or $7\ \frac{4}{7}$

> The rule for adding mixed numbers is to add the fractions separate from the whole number and then add the two answers.

There are times when the fractions will total a mixed number so we reduce to lowest terms and add the whole number in with the whole numbers.

Example:

$$\begin{array}{rr} 13\ \frac{4}{9} & \frac{8}{18} \\ +\ 73\ \frac{5}{6} & +\ \frac{15}{18} \\ \hline 86 + & \frac{23}{18} \text{ or } 1\ \frac{5}{18} \end{array}$$

$$\begin{array}{r} 86 \\ +\ 1\ \frac{5}{18} \\ \hline 87\ \frac{5}{18} \end{array}$$

Practice Problems 2.20 - 2.21

Do these Exercises:

20)

$$\begin{array}{r} 79\ \frac{7}{12} \\ +\ 13\ \frac{7}{9} \\ \hline \end{array}$$

21)

$$\begin{array}{r} 45\ \frac{3}{8} \\ +\ 17\ \frac{15}{16} \\ \hline \end{array}$$

B. SUBTRACTION OF FRACTIONS

(The same rules which applied to addition will also apply to subtraction of fractions and these are:)

When combining fractions by addition or subtraction:

1) The denominator of the fractions must be identical.

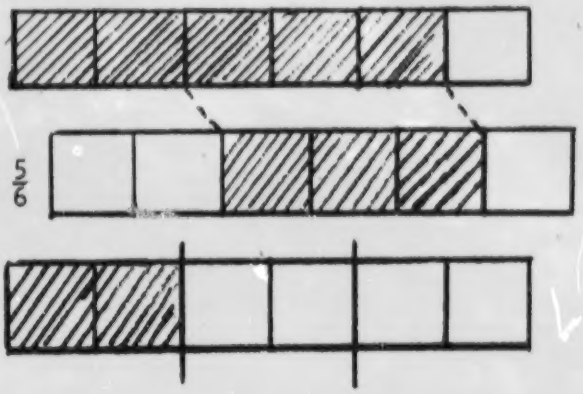
2) The numerators are then combined by addition or subtraction to obtain the new numerator.

3) The new denominator is the same as the like denominators of the original fractions.

When subtracting like fractions we really subtract the numerators and keep the denominator which is common.

Let us subtract $\frac{3}{6}$ from $\frac{5}{6}$

We have a chart



The shaded area represents $\frac{5}{6}$

If we take away $\frac{3}{6}$

We would have $\frac{2}{6}$ or $\frac{1}{3}$

or

Another example: $\frac{11}{18} - \frac{5}{18} = \frac{6}{18}$ or in Lowest terms $\frac{1}{3}$

Practice Problems 2.22 - 2.24

Subtract the following examples:

22) $\frac{13}{16} - \frac{3}{16} =$ _____

23) $\frac{10}{13} - \frac{7}{13} =$ _____

24) $\frac{41}{50} - \frac{3}{50} =$ _____

When subtracting with unlike fractions, we follow the same rule as adding unlike fractions. We must first find the common denominator and express the fractions in relation to this common denominator.

Example: $\frac{5}{6} - \frac{4}{9}$

common denom. $\frac{15}{18} - \frac{8}{18}$

Now we subtract the numerators and keep the common denominator giving us $\frac{7}{18}$

The most common way to write subtraction of fractions is as follows:

$$\begin{array}{r} \frac{5}{6} = \frac{15}{18} \\ -\ \frac{4}{9} = \frac{8}{18} \\ \hline \frac{7}{18} \end{array}$$

Practice Problems 2.25 - 2.26

Subtract the following:

25) $\begin{array}{r} \frac{3}{4} \\ -\ \frac{1}{16} \\ \hline \end{array}$

26) $\begin{array}{r} \frac{7}{8} \\ -\ \frac{1}{6} \\ \hline \end{array}$

When subtracting mixed numerals we subtract using the same rules as we did when adding mixed numerals. First convert the fractions to fractions with a common denominator, subtract the fractions, subtract the whole numbers, and combine the whole number and fractions for the answer.

Example: $26\ \frac{13}{20}$ or $26 + \frac{13}{20}$

$-12\ \frac{3}{5}$ $-12 + \frac{12}{20}$

$$\begin{array}{rrr} 26 & + & \frac{13}{20} \\ -\ 12 & + & \frac{12}{20} \\ \hline 14 & + & \frac{1}{20} \end{array}$$

$= 14\ \frac{1}{20}$

Practice Problems 2.27 - 2.28

Do the following subtraction:

27) $\begin{array}{r} 5\ \frac{3}{5} \\ -\ 2\ \frac{1}{4} \\ \hline \end{array}$

28) $\begin{array}{r} 53\ \frac{7}{12} \\ -\ 12\ \frac{3}{8} \\ \hline \end{array}$

When we subtract Mixed Numbers we sometimes have a problem much like subtracting whole numbers.

If we try to subtract the following: $7\frac{1}{6} - 2\frac{5}{6}$

$$\begin{array}{r} 6/\!\!1\frac{1}{6} + \frac{6}{6} = \frac{7}{6} \\ -\ 2\frac{5}{6} \quad \rightarrow \quad \frac{5}{6} \\ \hline 4 \qquad\qquad \frac{2}{6} \end{array}$$

In the fractional portion of the problem we have a common denominator, but we can't take 5 from 1 in the numerator, so we borrow a whole 1 from the 7 leaving 6.

Then we express the borrowed 1 as $\frac{6}{6}$ and add that to the $\frac{1}{6}$ giving us $\frac{7}{6}$.

or $4\frac{1}{3}$

Now we can subtract $\frac{5}{6}$ from $\frac{7}{6} = \frac{2}{6}$ or $\frac{1}{3}$.

Subtract 2 from 6 = 4

Adding 4 and $\frac{1}{3}$ giving a total of $4\frac{1}{3}$.

Practice Problems 2.29-2.31

Now with this principle of borrowing a whole one and expressing in a fraction, do the following subtractions.

29) $9\frac{2}{7} - \frac{3}{7}$

30) $9\frac{3}{8} - 4\frac{15}{16}$

31) $25\frac{3}{5} - 14\frac{2}{3}$

C. MULTIPLICATION OF FRACTIONS

When one of the factors is a whole number, such as $\frac{2}{3} \times 2$, we multiply the whole number by the numerator of the fraction and put the product over the denominator.

Example: $\frac{2}{3} \times 2 = \frac{4}{3}$

Then we convert this to a mixed number by dividing the denominator into the numerator.

Example: $3/\overline{4}$ (1 with remainder 1) $= 1\frac{1}{3}$ is the Mixed Number.

To use a graphic picture of this procedure:

$\frac{2}{3}$ x2 = $\frac{4}{3}$ or $1\frac{1}{3}$

Practice Problems 2.32 - 2.34

Now you should be able to do these exercises. Don't forget to express the product in the simplest form.

32) $3 \times \frac{3}{11} =$

33) $5 \times \frac{3}{7} =$

34) $5 \times \frac{1}{10} =$

When both the numerals are expressed in fractions, the rule is to multiply the numerators, then multiply the denominators, giving you a new numerator and denominator.

Example: $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

Sometimes the product can be reduced to simpler terms. When it can be reduced it should be reduced.

Example: $\frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$

Now with this principle known we can do the following multiplications of fractions:

Practice Problems 2.35 - 2.37

35) $\frac{6}{5} \times \frac{3}{5} =$

36) $\frac{1}{3} \times \frac{6}{9}$

37) $\frac{8}{9}$
$\times \frac{5}{7}$

To multiply a mixed number by a whole number the rules states to first multiply the whole number by the whole number part of the mixed number. Then we multiply the whole number by the numerator of the fractional part of the mixed number, then add the two.

Example:

$$9 \times 4\frac{1}{6} = 9 \times 4 + 9 \times \frac{1}{6} = 36 + \frac{9}{6} = 36 + 1\frac{3}{6}$$

$$= 36 + 1\tfrac{1}{2} = 37\tfrac{1}{2}$$

or

$$\begin{array}{r} 4\frac{1}{6} \\ \times\ 9 \\ \hline \end{array} = \begin{array}{r} 4 + \frac{1}{6} \\ \times\ 9 \\ \hline 36 + \frac{9}{6} \end{array} = 36 + 1\tfrac{1}{2} = 37\tfrac{1}{2}$$

With the above example you can see we have two multiplications and one addition to do to solve the problem. If we were to multiply two mixed numbers we would have to do the process twice which makes six operations to solve the problem. Large numbers would make the procedure very cumbersome.

If we were to change all numbers into fractions (proper or improper) and multiply the fractions, the procedure would be much simpler.

Example:

$$1\frac{2}{3} \times 4\frac{1}{5}$$

$$\frac{5}{3} \times \frac{21}{5} = \frac{105}{15} \div 15 = \frac{7}{1} = 7$$

Example:

$$5 \times 2\frac{1}{5}$$

$$\frac{5}{1} \times \frac{11}{5} = \frac{55}{5} \div 5 = \frac{11}{1} = 11$$

The only remaining problem with this new system is the simplifying of the fraction.

One way to lessen this problem is by what we call cross division or canceling out.

The rule states that with the multiplication of fractions we can divide the numerator of one fraction and the denominator of the other fraction with a number and not change the value of the fractions as a whole. We can simplify the factors of a fraction in the same way:

We divided both numerator and denominator by 5.

$$\frac{1}{\cancel{5}_{1}} \times \frac{\cancel{10}^{2}}{3} = \frac{1}{1} \times \frac{2}{3} = \frac{2}{3}$$

We divide the 3 and the 21 by 3 and the 5 and the 5 by 5.

$$\frac{1}{\cancel{5}_{1}} \times \frac{\cancel{21}^{7}}{\cancel{5}_{1}} = \frac{1}{1} \times \frac{7}{1} = \frac{7}{1} \text{ or } 7$$

We divide the 5 and the 10 by 5 and the 2 and the 4 by 2.

$$\frac{\cancel{5}^{1}}{\cancel{10}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{4}_{2}} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

The process is to change any whole numbers into fractions by putting the number over 1. Then change any mixed numbers to improper fractions. Next, use cross division or canceling out and then multiply the numerators and multiply denominators. Last, if necessary, simplify to lowest terms or change to a mixed number.

Example: $2\frac{2}{3} \times 6\frac{1}{4} = \frac{\cancel{8}^{2}}{3} \times \frac{25}{\cancel{4}_{1}} = \frac{50}{3} = 16\frac{2}{3}$

Practice Problems 2.38 - 2.40

38) $3\frac{3}{4} \times 1\frac{3}{5} =$ _____

39) $\frac{2}{3} \times 3\frac{1}{5} =$ _____

40) $1\frac{2}{9}$

$\times \frac{3}{22}$ = _____

D, DIVISION OF FRACTIONS

In a division problem with fractions, we have a dividend which will be divided by a <u>divisor</u> to give you a <u>quotient</u>.

Example: $\frac{1}{2} \div \frac{4}{5} = \frac{\frac{1}{2}}{\frac{4}{5}}$

To divide $\frac{1}{2}$ by $\frac{4}{5}$ we first find the reciprocal of the divisor $\frac{4}{5}$.

> The reciprocal of any fraction is the fraction turned upside down or inverted.

Example: $\frac{4}{5}$ has a reciprocal of $\frac{5}{4}$

To divide $\frac{1}{2}$ by $\frac{4}{5}$ we simply invert $\frac{4}{5}$ (turn it upside down) and multiply as usual.

$$\frac{1}{2} \div \frac{4}{5} \left(\frac{\frac{1}{2}}{\frac{4}{5}} \right) \quad \frac{1}{2} \times \frac{5}{4} = \frac{5}{8}$$

<u>Remember we follow all the rules of multiplying fractions when we do this.</u>

Example : $\frac{2}{3} \div \frac{3}{4}$

<u>Invert and multiply</u> $\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$

Example: $\frac{9}{16} \div \frac{15}{8}$

<u>Invert and multiply</u> $\frac{\cancel{3}}{\cancel{9}} \times \frac{1}{\cancel{8}} = \frac{3}{10}$

(use canceling) $\cancel{16}\ 2 \quad \cancel{15}\ 5$

Example: $1\frac{13}{15} \div 6\frac{2}{5}$

$\frac{28}{15} \div \frac{32}{5}$

<u>Change to improper fractions.</u>

$\frac{\cancel{28}^{7}}{\cancel{15}_{3}} \times \frac{1}{\cancel{32}_{8}} = \frac{7}{24}$

<u>Invert the divisor and cancel out.</u>

Students Name: _____ Date: _____

Chapter II Assignment

(This assignment is to be returned to instructor)

Change these improper fractions to mixed numbers:

1. $\frac{13}{8}$ = 2. $\frac{25}{7}$ = 3. $\frac{14}{4}$ = 4. $\frac{7}{3}$ = 5. $\frac{28}{6}$ =

Reduce these fractions to lowest terms:

6. $\frac{15}{45}$ = 7. $\frac{7}{21}$ = 8. $\frac{216}{324}$ = 9. $\frac{176}{42}$ = 10. $\frac{27}{72}$ = 11. $\frac{75}{105}$ =

Add

12. $\frac{1}{2} + \frac{1}{5} + \frac{3}{5} + \frac{2}{5}$

13. $\frac{3}{12} + \frac{7}{10} + \frac{9}{5}$

14. $\frac{1}{6} + \frac{3}{12} + \frac{9}{32}$

15. $\frac{2}{3} + \frac{1}{18} + \frac{1}{9}$

16. $11\frac{1}{3} + 2\frac{3}{4} + \frac{5}{6}$

Subtract

17. $\frac{2}{3} - \frac{4}{9}$

18. $\frac{3}{5} - \frac{7}{12}$

19. $\frac{1}{12} - \frac{1}{25}$

20. $9\frac{1}{6} - 5\frac{7}{8}$

21. $\frac{7}{10} - \frac{1}{6}$

Multiply

22. $\frac{1}{5} \times \frac{3}{7}$ = 23. $\frac{7}{10} \times \frac{6}{2}$ = 24. $\frac{5}{7} \times \frac{3}{4}$ = 25. $\frac{5}{9} \times \frac{3}{5}$ =

26. $\frac{1}{6} \times \frac{9}{32}$ =

Divide

27. $\frac{1}{7} \div \frac{3}{5}$ 28. $\frac{1}{6} \div \frac{7}{10}$ 29. $3 \div \frac{7}{5}$ 30. $\frac{2}{3} \div \frac{10}{21}$

31. $\frac{1}{2} \div \frac{5}{6}$

(OVER)

Preparations for Willing Water Week are being made. A rope will be used to guide the people through the plant. The following measurements (in yards) were made. What is the total length of rope needed?

$17\frac{1}{2}$	$9\ \frac{1}{6}$	$10\ \frac{1}{8}$
$135\frac{5}{8}$	$44\ \frac{3}{4}$	$27\ \frac{5}{6}$
14	$12\ \frac{1}{4}$	$17\ \frac{3}{8}$
$282\frac{2}{3}$	$20\ \frac{1}{3}$	$6\ \frac{7}{8}$

Show all calculations:

A) How many yards of rope are required expressed as an improper fraction?

B) How many yards of rope are required expressed as a mixed number?

SECTION I CHAPTER III

DECIMALS

In our number system the regular whole numbers are grouped in ones, tens, hundreds, thousands, and so on, increasing in multiples of 10 as we move left from the decimal point.

Example:

1. = ones
10. = tens
100. = hundreds
1000. = thousands
etc.

The fractions of numbers are grouped to the right of the decimal point and decrease in multiples of tenths as we move to the right.

Example:

$.1 = \frac{1}{10}$ = tenths

$.01 = \frac{1}{100}$ = hundredths

$.001 = \frac{1}{1000}$ = thousandths

etc.

The numbers to the left of the decimal point are the whole numbers and those to the right of the decimal point are the fractional parts.

Example:

2.5
two and five tenths

$2 + \frac{5}{10}$

Example:

16.34
sixteen and thirty four hundredths

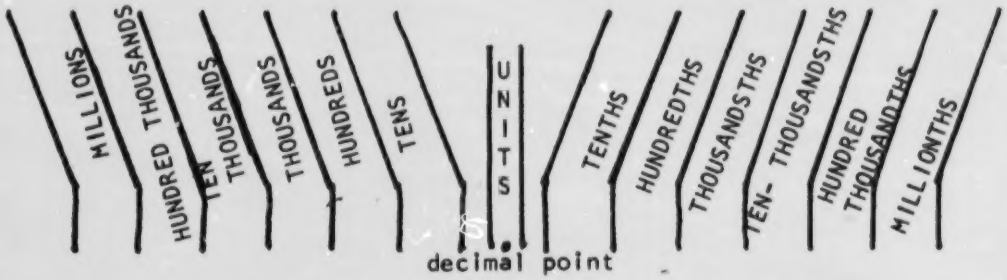
$16 + \frac{34}{100}$

Example:

321.296
Three hundred twenty one and two hundred ninety six thousandths

$321 + \frac{296}{1000}$

The following chart names the six places to the left and to the right of the decimal point.



A. ADDITION OF DECIMALS

In the next two sections of this chapter it is very important that you line up the decimal points under each other, before attempting the addition and/or subtraction of the section,

When we add whole numbers we put all the ones in a column, all the tens in the next to the left, and so on. Then it is easy to add all the ones, tens, etc.

Example: Add - 5 + 216 + 23 + 642 =

$$\begin{array}{r} +\ \ 5 \\ +\ 216 \\ +\ \ 23 \\ +\ 642 \\ \hline 886 \end{array}$$

In decimals we do the same thing with all the tenths, hundredths, thousandths, etc., and the decimal points.

Example: Add - 0.3 + .0592 + .167 + .203 =

$$\begin{array}{r} +\ \ .3 \\ +\ .0592 \\ +\ .167 \\ +\ .203 \\ \hline .7292 \end{array}$$

To make the addition even easier we add zeros before whole numbers and after decimal numbers to square up the problem. These do not change the value of the numbers.

Then we add in the usual way but remembering to include the decimal point in the answer in the same column as the other decimal points.

Example:

$$\begin{array}{r} 02.859 \\ 25.980 \\ 04.790 \\ +\ 05.400 \\ \hline 39.029 \end{array}$$

Practice Problems 3.1 - 3.3

1) 13.6 + 9.7 + 3.5 + 10.4 =

ans._____

2) 9.5 + .016 + 32.3 + 7.0312 =

ans._____

3) 0.3
4.0
0.7
5.0
0.6
+ 8.0

ans._____

B. SUBTRACTION OF DECIMALS

The same rules we used for addition are used in subtraction so long as you keep the columns in line and the decimals in line.

The same rules of borrowing for subtraction of whole numbers will still apply in decimal subtraction.

Example: 3 2
4.365
- 2.780
1.585

Practice Problems 3.4 - 3.6

4) 4.9
- 3.0

ans._____

5) 39.1
- 5.431

ans._____

6) 17.369 - 4.29 =

ans._____

C. MULTIPLICATION OF DECIMALS

In multiplication with whole numbers we went over the principle of the powers of 10. Any number multiplied by ten would be that number plus a 0 on the right end.

Example: 35
x 10
350

Every whole number has an unwritten decimal point to the right. Thus, when we multiply by ten we are really moving the decimal point one place to the right.

Example with decimals:
$$\begin{array}{r} 35. \\ \times\ 10 \\ \hline 350. \end{array}$$

This same principle still applies no matter where the decimal point is located.

Example: $32.6 \times 10 = 326.$

Example: $326.597 \times 10 = 3265.97$

Example: $0.0621 \times 10 = 0.621$

When we multiply by 100 we move the decimal point two places to the right.

Example: $3.7632 \times 100 = 376.32$

Example: $.01623 \times 100 = 1.623$

This goes on through the millions if necessary.

Practice Problems 3.7 - 3.9

7) $\begin{array}{r} 43 \\ \times\ 10 \\ \hline \end{array}$ 8) $\begin{array}{r} .35 \\ \times\ 10 \\ \hline \end{array}$ 9) $\begin{array}{r} 356.1 \\ \times\ 100 \\ \hline \end{array}$

We will now multiply decimal numbers by decimal numbers.

In this we do not have to keep the decimals in a column. We proceed to multiply just as we do with whole numbers and put in the decimal point (in the answer) later, by counting the number of decimal places involved.

Example:
$$\begin{array}{r} 1.17 \\ \times\ 2.4 \\ \hline 468 \\ 234 \\ \hline 2808 \end{array}$$

Now to find where the decimal point should be. There are two places to the right of the decimal point in 1.17 ie .17 or .$\underline{x}$ $\underline{x}$ and one place to the right of the decimal point in 2.4 ie .4 or .$\underline{x}$. The sum of these places 2 (.$\underline{x}$ $\underline{x}$) + 1 (.$\underline{x}$) is three and we must insert the decimal point three places to the left of the last digit.

$$2808 = 2.808$$

Example:	16.3	.x	1 decimal place
	x 2.104	+ .x x x	3 decimal places
	652		
	1630		
	326		
	342952	.x x x x	4 decimal places

342952 = 34.2952

Practice Problems 3.10 - 3.12

10) .809 x 9.15

11) 7.3 x .9

12) 7.71 x 3.9

D. DIVISION OF DECIMALS

When we divide any number by 10, 100, 1,000, etc., we use the tens principle as in multiplication except we move the decimal to the left.

Example: 341 ÷ 10 = 34.1

341 ÷ 100 = 3.41

341 ÷ 1000 = .341

3.41 ÷ 10 = .341

.0341 ÷ 100 = .000341

Practice Problems 3.13 - 3.16

13) 64.3 ÷ 100 =

14) .146 ÷ 10 =

15) 4 ÷ 1,000 =

16) 465.1 ÷ 100 =

Now we will do Division with Decimals when the divisor is a whole number.

This is done the same as all whole numbers except we have to be sure to put the decimal in the answer.

Example:

```
          7.19      Quotient
Divisor  9/ 64.71    Dividend
         63
         17
          9
         81
         81
          0
```

TIP: The best way to do this is to put the decimal in the quotient directly above the decimal in the dividend before doing the numerical division.

Practice Problems 3.17 - 3.19

17) $7\overline{)34.86}$ 18) $21\overline{)640.5}$ 19) $45\overline{)188.1}$

Now for division when both the divisor and dividend are expressed in decimals.

Example:	$\frac{2.10}{0.06}$	or	$.06\overline{)2.10}$

The procedure here is to move the decimal to the right in the divisor to make a whole number. At the same time we move the decimal in the dividend the same number of points to the right.

.06 2.10

Giving us $6\overline{)210}$ then divide. This does not change the problem at all but it does make it easier to solve.

The reason this is easier is because; $.06\overline{)2.10}$ can be expressed as a fraction.

$$\frac{2.10}{.06}$$

You can multiply both the numerator and denominator by 100 without changing the fraction.

$\frac{2.10 \times 100}{.06 \times 100}$ Giving us $\frac{210}{6}$ which is $6\overline{)210}$

With this multiplication by tens we can solve a decimal division problem the same way we did when the divisor was a whole number.

Just remember to multiply the divisor and dividend by the same power of ten.

Example of moving decimal. (or multiply)

$3.569\overline{)534.769.5}$

giving us: $3569.\overline{)534769.5}$

Practice Problems 3.20 - 3.22

20) $.12\overline{)47.64}$ 21) $.28\overline{).8876}$ 22) $3.7\overline{)1494.8}$

E. DECIMALS TO FRACTIONS

A fraction is a division problem.

$\frac{1}{3}$ means $3\overline{)1.000}$
one divided by three.

If you complete the division of any fraction you will get the decimal equivalent of that fraction.

Example: $\frac{2}{5}$ is $5\overline{)2.0}$ = .4

Example: $\frac{15}{20}$ is $20\overline{)15.00}$ = .75
140
100
100

Practice Problems 3.23 - 3.25

23) $\frac{7}{8}$ = 24) $\frac{7}{25}$ = 25) $\frac{4}{625}$ =

We should also be able to convert the decimals to fractions. To do this we merely write the decimal as a fraction.

Example: .25 is $\frac{25}{100}$
Then we reduce to lowest terms.

$\frac{25}{100} \div 25 = \frac{1}{4}$

Example: .341 is $\frac{341}{1000}$

Practice Problems 3.26 - 3.28

Convert these decimals to fractions:

26) .2 = 27) .75 = 28) .4 =

F. ROUNDING OFF DECIMALS

The process of rounding off numbers is one that requires thought and experience. In most applied problems the decimals seldom need to be carried beyond three places, BUT there are no hard and fast rules. When a certain decimal is given to five or six places you may have to use all five or six places. The type of calculations that you are making will determine the degree of accuracy that you must adopt. In every practical problem you must use common sense.

In rounding off a number you will drop one or more figures at the right. So if you wish to express 7.273 inches to the nearest tenth of an inch you write 7.3 inches, since 7.273 is closer to 7.3 than 7.2.

Another example is: 10.486 rounded off to the nearest tenth:

10.486 = 10.5 Since 10.486 is closer to 10.5 than 10.4.

Or how about 5.338 to the nearest tenth.

5.338 = 5.3 Since 5.338 is closer to 5.3 than 5.4.

In general, you can use the following method in rounding off a number to a specified number or decimal place:

1. Decide which figure is the specified one.
2. Glance at the next figure to the right of this one.
3. When the figure that you drop is 5 or greater increase the preceding figure by one. Where the figure that you drop is less than 5, keep the preceding figure as it is.

Example:

Round off the following numbers to two decimal places and add.

12.4783, 3.344, 20.16523, 110.6621

12.4783 = 12.48
3.344 = 3.34
20.16523 = 20.17
110.6621 = +110.66
146.65 ans.

Practice Problems 3.29 - 3.33

Round off to nearest two decimal places:

29) 33.6520 = _____ 30) 7.632 = _____

31) 3.336 = _____ 32) 12.7895 = _____

33) 10.7188 = _____

Date: _____

Students Name: _____

CHAPTERS III ASSIGNMENT SHEET

(to be returned to Instructor)

Chapter III - Decimals

Add:

1. 13.6 + 9.7 + 3.5 + 10.4 =

2. 9.5 + .016 + 32.3 + 7.0312 =

3. 0.3 + 4.0 + 0.7 + 5.0 + 0.6 + 8.0 =

4.
```
  431.916
   45
     .007
+   9.73
```

5. 3 + 2.1 + 4.09 + 6.093

Subtract:

6. 4.9 − 3.0

7. 39.1 − 5.431

8. 17.369 - 4.29 =

Multiply:

9. .809 × 9.15

10. 7.3 × .9

11. 7.71 × 3.9

12. 16.3 × 2.104

Divide:

13. 7/ 34.86

14. 21/ 640.5

15. 4.5 / 188.1

16. 12/ 47.64

17. .28/ .8876

(OVER)

Convert these decimals to fractions:

18. .2 =
19. .75 =
20. .4 =

Round off to the nearest tenth:

21. 14.697 _____
22. 3.98112 _____
23. 3.72 _____

Round off to the nearest hundredth:

24. 41.0951 _____
25. 3.8192 _____

SECTION I CHAPTER IV
PERCENTAGE

BASIC FACTS:

1. Percent uses a special symbol "%".

2. The sign % does the duty of 2 decimal points.

3. Percent comes from the Latin meaning "per 100."

   In other words we are comparing the number to 100.

   20% compares 20 to 100.

4. This comparison is called a ratio and we usually represent a ratio in the form of a fraction:

$$20\% = \frac{20}{100}$$

What are the Fractional equivalents of the following?

35% =

15% =

71% =

Per cent can also be expressed as a decimal.

Example:	$34\% = \frac{34}{100} = .34$

All three of these are the same thing expressed in a different way.
Four examples of this concept follow:

Percent	Fraction	Decimal
15%	$\frac{15}{100}$	.15
25%	$\frac{25}{100}$	.25
20%	$\frac{20}{100}$	.20
69%	$\frac{69}{100}$	.69

Some of these fractional expressions can be reduced to simpler terms as we did in the fraction section. This does not change the value of the fraction.

Examples: $\frac{20}{100} = \frac{2}{10} = \frac{1}{5}$

$\frac{30}{100} = \frac{3}{10}$

$\frac{25}{100} = \frac{1}{4}$

Practice Problems 4.1 - 4.3

Change these percentages to fractions and reduce to simplest terms.

1. 50% =

2. 10% =

3. 80% =

There are times when we have a percentage that is larger than 100. When you change that to a fraction you will get an improper fraction.

Example: $130\% = \frac{130}{100}$

This fraction should be reduced to $1\ \frac{30}{100}$

or further to $1\ \frac{3}{10}$

This reduced fraction expressed as a decimal will be $1\ \frac{3}{10} = 1.3$

Many people have trouble remembering the rules for "changing from percents to decimals and vice-versa." It may help you to remember that:

$1 = \frac{100}{100} = 100\%$

Therefore: $1.00 = 100\%$
$.01 = 1\%$

That is, we multiply a decimal by 100 to get a percent.

$1.00 = (1.00 \times 100)\% = 100\%$

We divide a percent by 100 to get a decimal.

$1\% = \frac{1}{100} = .01$

Practice Problems 4.4 - 4.6

Change these percentages to fractions, reduce and express as a decimal.

4. 131% =

5. 245% =

6. 161% =

If you are still not quite sure of how to change from a % to a decimal you can then convert a percentile directly to a decimal using a very simple procedure. What you do is take the percentile and move the decimal two places to the left. (this is dividing by 100)

Example: 21 % = .21

261 % = 2.61

If you want to convert a decimal to a percentile you merely do the reverse. We move the decimal two points to the right. (This is multiplying by 100)

Example: 4.65 = 465%

.05 = 5%

Now we can change % to decimal and decimal to %.

Practice Problems 4.7 - 4.8: Convert the following to a percentage.

7. .35 = ____ 8. 7.9 = ____

Practice Problems 4.9 - 4.10: Convert the following to a decimal.

9. 21% = ____ 10. 23% = _____

The question always comes up as to why we should be able to do these conversions. How is all this manipulation going to be used in solving problems?

To find what percent one number is of another it is best to write the two numbers as a fraction, convert the fraction to a decimal by division and then express the decimal in percent form.

Example:

What is 50% of 60 shear pins?

Note: "of" means multiply.

50% x 60 = ?

Convert to --- .50 x 60 = 30

or Convert to --- $\frac{1}{2}$ x 60 = 30

Example:

If we had 35 shear pins and used 5 shear pins for replacement, what % of shear pins did we use?

The fraction we arrive at is - $\frac{5}{35}$. Or we are comparing 5 shear pins to 35 shear pins.

$$\frac{5}{35} = \frac{1}{7} \text{ of the shear pins}$$

$\frac{1}{7}$ converted to decimal is $.14285^{+}$

(do the division $\frac{1.00000}{7} = .14285$)

Decimal converted to % = .14285 x 100 = 14.285%

Practice Problems 4.11 - 4.15

11) What is 5% of $60.00? Ans. _____

12) What is 20% of 320 days? Ans. _____

13) What is 15% of 225 pounds? Ans. _____

14) What is 110% of 50 grams? Ans. _____

15) What is 50% of 228 years? Ans. _____

STUDENTS NAME:_____ DATE:_____

CHAPTER IV

(To be returned to instructor)

1. Applying the concept of percentages to treatment plant problems, suppose a certain piece of equipment was having mechanical difficulties. Since percentage is often based on random sample, an operator might test the equipment 25 times and find that it failed 6 times of those 25 tests.

   What percentage failure does this represent?

2. A treatment plant operator has been ill during the past month and has had to miss a number of days work. If he misses an average of 3 days out of 7 days,

   What is his percentage absence?

3. The Planning Commission of a community voted on a bond issue concerning the construction of a new treatment plant. If 8 are in favor of the bond issue and 5 are opposed,

   What percent of the commission was in favor of the bond?

(OVER)

Complete the chart.

4-8.

Percent	How it is Read	Expressed as a ratio
35%		
	Six percent	
		$\frac{14}{100}$
215%		
		$\frac{100}{100}$

Express in fractions in their simplest form:

9. 45% =
10. 110% =
11. .15 =
12. .07 =

Express in decimals:

13. 43% =
14. 3.5% =
15. 1/5 =
16. 36/100 =

Express in percent:

17. .157 =
18. 3.8 =
19. $\frac{3}{25}$ =
20. $\frac{9}{100}$ =

MATHEMATICS FOR WATER SUPPLY OPERATORS

SECTION I CHAPTER V

RATIO AND PROPORTIONS

A ratio is the established relationship between two numbers and may be written as 1:2 or 1/2; 4:7 or 4/7.

When the relationship between two numbers of a ratio is the same as that between two other numbers in ratio, a proportion exists.

As we learned in previous lessons; a fraction is a ratio. We are comparing two numbers.

Example: $\frac{2}{5}$ -There are two partial portions
-There are five portions in the whole unit.

The whole is divided into 5 parts and we are representing 2 of those five parts with the fraction $\frac{2}{5}$.

A proportion is a statement that two ratios (or fractions) are equal.

For example: 2:5 = 4:10

This is expressed as: Two is to five as four is to ten.
When we write this as fractions we have:

$$\frac{2}{5} = \frac{4}{10}$$

If we cross multiply this we have proven the proportion (or proven the fraction is equal).

$$\frac{2}{5} \times \frac{4}{10}$$

$$20 = 20$$

With this concept of proportion, we are now ready to put the principle to a practical use.

If we can set up a proportion with one of the numbers missing (or unknown) we can then solve by cross multiplication.

Example:

If twelve bolts cost you $1.58, what will six bolts cost?

Set up as a proportion -------- $12:1.58 = 6:x$

Change to a fraction $\frac{12}{1.58} = \frac{6}{X}$

Cross multiply $12X = 6 \times 1.58$

Divide both sides by 12 $\frac{12X}{12} = \frac{9.48}{12}$

Solve for? $X = \$.79$ (for six bolts)

This example is very simple because we know 6 is just $\frac{1}{2}$ of 12 so we could take $\frac{1}{2}$ of 1.58 and have the answer. But if we were figuring in other numbers it could be more difficult.

Example:

We know 12 bolts would cost $1.58. How much would 126 bolts cost?

$12:1.58=126:x$ (use x as the unknown)

Change to a fraction $\frac{12}{1.58} = \frac{126}{x}$

Cross Multiply $12x = 126 \times 1.58$

Solve for x $\frac{12x}{12} = \frac{199.08}{12}$

$x = \$16.59$

Practice Problems 5.1 - 5.3

Solve for x:

1) $81 : x = 45:10$ ans. _____

2) $x : 7 = 350:10$ ans. _____

3. The directions to mix a particular polymer state:

Add 5 pounds of dry polymer to each 500 pounds of water.

If you are mixing a batch of 1000 gallons of water (8340 pounds) how many pounds of polymer do you need?

ans._____

Students Name: _____ Date: _____

(to be returned to instructor)

CHAPTER V - ASSIGNMENT SHEET

Do the following solving for "x"

1. 15 : x :: 35 : 7

2. We have $30.00 to spend on polymers and they cost $1.20 for 10 lbs. How many lbs. of polymer can we buy?

3. Are 3/7 and 42/98 proportionate (show all calculations).

4. 4.7 pounds of chlorine compound are dissolved in 45 gallons of water. A second solution with 32 gallons of water has 3.1 pounds of the chlorine compound dissolved in it. Do the two solutions have a similar concentration of the chlorine compound?

5. 5.4 pounds of calcium hypochlorite are dissolved in 52 gallons of water. If we desire a solution of the same concentration, how many pounds of sodium hypochlorite must be dissolved in 30 gallons of water?

SECTION I CHAPTER VI

AVERAGES

Averages can be very useful tools in evaluating performance and/or process in operations. It often becomes difficult to determine TRENDS in performances.

> The calculations of an average is a method by which we may group the information which is then represented by a single number.

Two calculations of this type are the arithmetic mean and the median.

A. ARITHMETIC MEAN:

It is the most commonly used measurement of averages, rather than the median. We must keep in mind when we evaluate information based on averages, that the average affects the general routine of the group and that it doesn't affect, at least not necessarily any one element in that group.

The arithmetic mean is done as follows:

$$\frac{\text{TOTAL OF ALL SAMPLES}}{\text{NUMBER OF SAMPLES}}$$

Example:

The water temperature in a filter plant was recorded as follows: What was the average daily temperature?

1.	Monday	0° C
2.	Tuesday	5° C
3.	Wednesday	12° C
4.	Thursday	10° C
5.	Friday	16° C
6.	Saturday	16° C
7.	Sunday	9° C
	Total	68° C

$$\text{Arithmetic Mean} = \frac{\text{Total of All Samples}}{\text{Number of Samples}}$$

$$\text{Arithmetic Mean} = \frac{68}{7}$$

Arithmetic Mean = 9.7° ave. degrees centigrade

Example

The numbers of sick days taken during the year by each of 5 operators was recorded as follows: Operator A - 6 days, Operator B - 3 days, Operator C - 7 days, Operator D - 0 day, Operator E - 1 day.

For these five men, what was the average days of sick leave required?

1.	Operator A	6 days sick leave
2.	Operator B	3 days sick leave
3.	Operator C	7 days sick leave
4.	Operator D	0 days sick leave
5.	Operator E	1 day sick leave
	Total	17 days sick leave

$$\text{Arithmetic Mean} = \frac{\text{Total of all Samples}}{\text{Number of Samples}}$$

$$= \frac{17}{5}$$

$$= 3.4 \text{ days sick leave}$$

Practice Problems 6.1 - 6.2 - Calculate the arithmetic average.

1) Daily Turbidity was

Day 1 = 1.1
Day 2 = 0.8
Day 3 = 0.5
Day 4 = 1.2
Day 5 = 0.8
Day 6 = 0.7
Day 7 = 0.5

The weekly average was =

ans._____

2) The following pH's were recorded

4/1 = 6.8
4/8 = 6.5
4/15 = 6.7
4/22 = 7.0
4/29 = 6.9

The monthly average was =

ans._____

B. MEDIAN

The median is another type of average and is just about the opposite of the arithmetic mean in that there is little or no mathematics calculation involved.

The median average is the middle value of all samples taken.

The reason for this is because 50% of the sample values will be above the median and 50% will be below.

To do a median average we must first arrange our sample values in ascending or descending order. Once this is done, finding the middle value will not be difficult.

Example

The gpm (gallon per minute) flow to the clearwell was recorded every two hours for the following hours:

6 AM - 200 gpm
8 AM - 212 gpm
10 AM - 223 gpm
12 Noon - 219 gpm

What is the median gpm flow during this time span?

First Step: Arrange in ascending or descending order or rank.

1. 200 gpm
2. 212 gpm
3. 319 gpm
4. 223 gpm

In this case an even number of values, the second and third values represent the middle values. The median will be halfway between these two values. We can solve this by either of two methods.

Method #1

$$\begin{array}{r} 212 \\ +\ 219 \\ \hline 431 \end{array} \qquad \frac{431}{2} = 215.5 \text{ gpm median}$$

Method #2

$$\begin{array}{r} 219 \\ -\ 212 \\ \hline 7 \end{array} \text{ span} = 3.5 \text{ of mid span}$$

$$\begin{array}{r} 212.0 \\ +\ \ 3.5 \\ \hline 215.5 \end{array} \text{ gpm/median}$$

Practice Problems 6.3 - 6.4

Calculate the median of the following values:

3) Chlorine added per day
Day 1 = 5.4 pounds
Day 2 = 5.8 pounds
Day 3 = 6.2 pounds
Day 4 = 5.7 pounds
Day 5 = 5.9 pounds
Day 6 = 5.6 pounds

Ans. _____

4) Time of day at which peak demand occurred:

Day 1 - 10:05 AM
Day 2 - 10:10 AM
Day 3 - 9:45 AM
Day 4 - 9:30 AM
Day 5 - 9:20 AM
Day 6 - 9:35 AM

Ans. _____

Date: _____

Students Name: _____

CHAPTER VI - AVERAGES - ASSIGNMENT SHEET

(to be returned to instructor)

1. A water plant has ten electric motors having the following capacities: 30, 20, 75, 50, 100, 40, 25, 10, 15, 50 horsepower. Find the arithmetic mean for the ten electric motors in horsepower.

2. A plant flow meter registers the following flows in m.g.d.: .135, .126, .226, .187, .158, .200, .193.

   A) Find the arithmetic mean for the flows in mgd.
   B) What is the median flow?

3. A water treatment plant laboratory recorded the following fluoride concentrations in its water: 0.4, 0.8, 0.7, 1.0, 0.8, 1.1, 1.2, mg/l.

   A) What is the fluoride median of the water in mg/l?
   B) What is the arithmetic mean of the fluoride in mg/l?

SECTION I CHAPTER VII

MEASUREMENT

A. LINES AND DISTANCES

There are two standard systems of measure used in water treatment. They are the English system and the Metric system. The English system is widely used and is the one with which we are all familiar. The metric system has always been used in the laboratory and is now being adopted as the world standard of measure.

Two commonly used comparisons of measure are the inch in the English system and the centimeter in the Metric system. Comparing the two standards the inch is approximately $2\frac{1}{2}$ times as long as the centimeter.

In order to be really useful, a unit of measurement must be fixed, or made standard. When we say we are making a measurement, we mean we are comparing the quantity to be measured with a unit of measurement.

> Using the above we can say that our definition of measurement is: using a fixed standard to compare the quantity to be measured with a unit of measurement.

To illustrate this we are going to measure the length of lines. These lines may be straight or curved. They may be up, down, across, over, under, through, or around. These lines are called by different names; some of which are length, width, depth, perimeter, radius, diameter, and circumference.

To do the measuring in this exercise, each student will need to have a ruler with both English and Metric units of measures.

Practice Problems 7.1 - 7.4

Measure the following line segments:

Using Inches

1) A._____B. ans._____

2) A._____B. ans._____

Using Centimeters

3) A._____A. ans._____

4) A._____B. ans._____

B. PERIMETERS AND AREAS

Linear distances are important by themselves. We need to know many different distances. For example: How much water pipe do we need? How much fencing do we need to surround and protect our plant?

We are not only interested in the length of a treatment unit but also in the distances around it.

> The distance around a triangular or rectangular unit is called the perimeter. The perimeter or distance around a circular unit has a special name and is called the circumference.

We also have an interest in area measurement.

> Area measurement is a measurement of the amount of space on the surface of an object.

We must be concerned with the measurement of surface areas. For the purpose of processing we will limit this discussion to areas of flat surfaces. We need to know: How much surface do we have for settling? How much surface do we have on out filter?

Remember distances are measured in linear units i.e. inches (in.), feet (ft.), centimeters (cm), meters (m), etc. (Remember also, that in actual practice measurements are only approximations).

Areas are measured in square units i.e. square inches (in^2), square centimeters (cm^2), square feet ($ft.^2$).*

*The little superscript or exponent ^{2} means to multiply by itself.

For example:

ft^2 = ft x ft = feet x feet
r^2 = r x r = radius x radius
D^2 = D x D = Diameter x Diameter

The surface of the structures that we will study are in one plane or in other words have a flat surface.

*Remember then that:

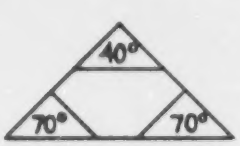
The distance around this surface is called perimeter or, in the case of a circle, it is called the circumference, and that area is related to the surface of an object.

1. TRIANGLES

a. Types

General - All triangles have three sides and three angles. The size of the inside angles always adds up to 180^0.

For example:



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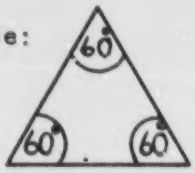
Under this general heading we have triangles which have special qualifications. These are given descriptive names.

Equilateral Triangles

Special qualifications: All three sides are equal and all three angles are each 60° and equal.

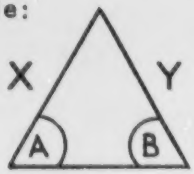
For example:



Isosceles Triangle

Special Qualifications: The two sides are equal and the opposite angles are equal.

For example:



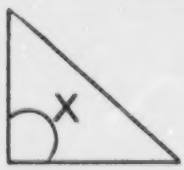
X & Y are equal

a & b are equal

Right Triangle

Special Qualifications: One of the angles is a 90° or right angle.

For example:



X is 90° or a right angle

Scalene Triangle

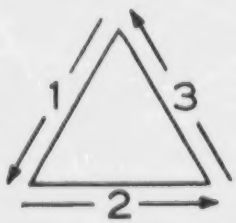
This is the name given to a triangle that doesn't meet any special qualifications. This is one with none of the sides or angles equal.

b. Perimeter of Triangles

The perimeter of a triangle is the distance around the figure or the sum of the lengths of the three sides.

Formula: Perimeter = Side 1 + Side 2 + Side 3.

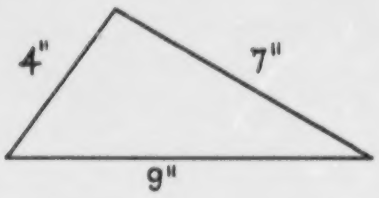
$$P = S_1 + S_2 + S_3$$



This formula applies to all triangles.

Example:

$P = S_1 + S_2 + S_3$

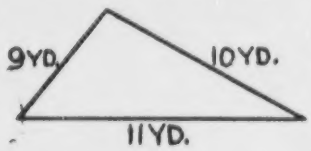


P = 4 in + 7 in + 9 in
P = 20 in

Practice Problems 7.5 - 7.7

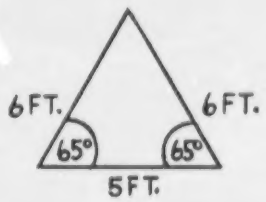
Find the perimeter of these t iangles:

5)



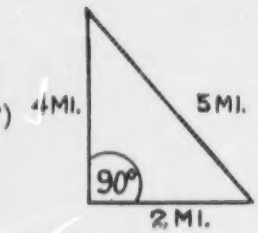
Ans. _____

6)



Ans. _____

7)



Ans. _____

C. AREA OF TRIANGLES

We also may need to know the area of a triangle. This area is the number of square units contained within the triangle. Only the actual length of one side of a triangle is of importance. It is chosen as the base, rotated to the horizontal position and the height of the triangle is then measured vertically from that base.

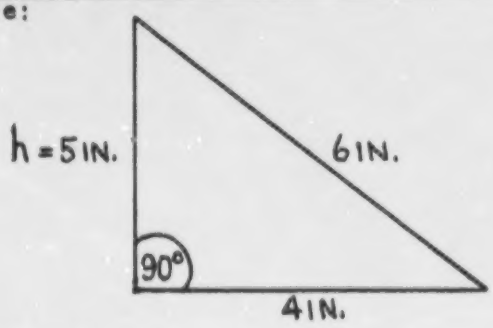
We do not have a simple way to measure area directly. Using dimensions which we can measure and a formula (formulas are directions or instructions) we can calculate the area.

The area of a triangle is the base times the height divided by 2.

Formula : $\text{Area} = \frac{\text{base} \times \text{height}}{2}$

$A = \frac{b \times h}{2}$

Example:



$A = \frac{b \times h}{2}$

$A = \frac{4\ in \times 5\ in}{2}$

$A = \frac{20\ in^2}{2}$

$A = 10\ in^2$

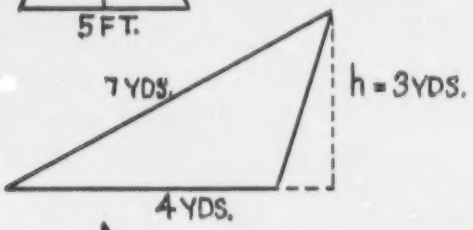
Practice Problems 7.8 - 7.10

Find the area of these triangles:

8)

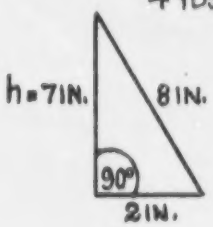
Ans. _____

9)



Ans. _____

10)



Ans. _____

2. QUADRILATERALS

A. Types

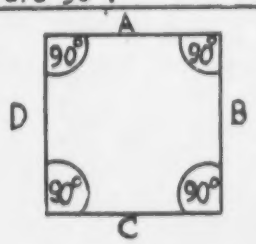
A quadrilateral is any figure that has four sides and four angles. The inside angles add up to 360°.

If there is a relationship between some of the angles and sides we give these figures special names.

Square

A square is a quadrilateral in which; all four sides are equal and all four angles are 90°.

For example:



All angles are 90° and equal.
All sides (A,B,C,D) are equal.

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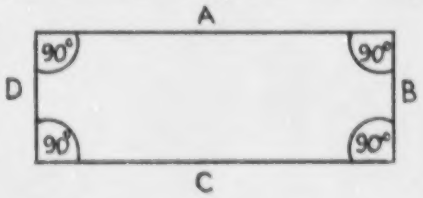
Rectangle

The rectangle is a quadrilateral in which; all four angles are 90° and the opposite sides are parallel and equal in length.

For example:



All angles are 90° and equal
side a + c are equal
side b + d are equal
but side a is not equal to b or d.

Like any plane figure, we should be able to find out two things about these figures. 1. The perimeter or distance around and 2. The area which is the square units enclosed by the sides.

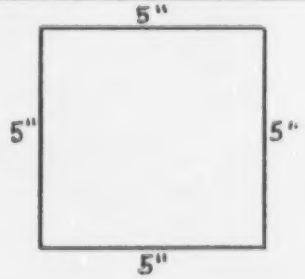
b. The perimeter of a Quadrilateral

The perimeter of a quadrilateral is the total length of all four sides.

The formula for this is $P = S_1 + S_2 + S_3 + S_4$

This formula works on all types of quadrilaterals.

Example: .

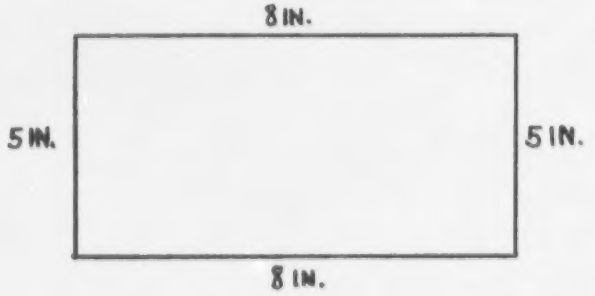


$P = S_1 + S_2 + S_3 + S_4$

$P = 5 + 5 + 5 + 5$

$P = 20$ inches

Example:



$P = S_1 + S_2 + S_3 + S_4$

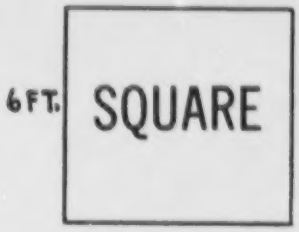
$P = 5 + 8 + 5 + 8$

$P = 26$ inches

Practice Problems 7.11 - 7.12

Find the perimeter of these quadrilaterals.

11)



ans. _____

12)

ans. _____

C. The area of a Quadrilateral

We must also be able to find the areas of quadrilaterals. The formulas change with the shape of the quadrilaterals. However, since we are concerned with squares and rectangles, the formulas are as follows:

Area of a square = $A = S^2$
Area of a rectangle = L x W

Square

Since we know a square has four equal sides and all angles are equal, we simply square the length of the side and get the Area. (area is always expressed in square units)

Example:

$A = S^2$
$A = 6 \text{ ft} \times 6 \text{ ft}$
$A = 36 \text{ ft}^2$

Practice Problems 7.13 - 7.14: Find the area of these squares:

13)

Ans. _____

14)

Ans. _____

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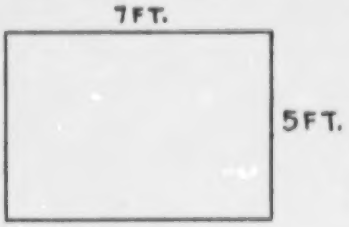
Rectangle

A rectangle is a four sided figure having four equal angles.

The rectangle has two different lengths of sides. One side is called the length and the other is called the width. If we multiply these two we come up with square units of the figure.

Example:

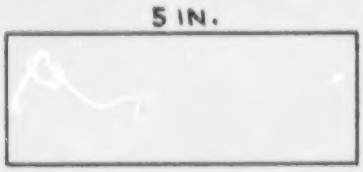


Area = L x W

A = L x W
A = 7 ft x 5 ft
A = 35 ft.

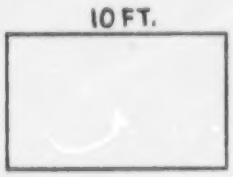
Practice Problems 7.15 - 7.16 Find the Area of these rectangles

15)



Ans. _____

16)



Ans. _____

3. CIRCLES

A Circle is a figure bounded by a curved line, every point of which is the same distance from another point, called the center. The curved ling or outer boundary of the circle is called the circumference.

Example:

A,B,C,D,E, & F are of equal length.

The distance from the center point to a point on the outside is called the radius. The radius of a circle is usually indicated by R.

The straight distance from one side of a circle to the other passing through the center is called the diameter. The diameter is always twice the radius and a diameter always divides a circle into 2 equal parts called semicircles or hemispheres.

Example:

D = 2 r

We have a mathematical constant that we use with circles. This is called pi (the symbol is π). π never changes in value and is the circumference (distance around the outside or perimeter) of a circle divided by the diameter of the circle.

$$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

This may be expressed as a fraction or as a decimal.

$\frac{22}{7}$ or 3.1416

Pi (π) will always = 3.1416

A. Circumference of a Circle

The formula is $C = 2\pi r$ or πD

This formula means 2 times π (3.14) times R (the radius) or π (3.14) times the diameter.

Circumference = 2π radius or π Diameter

$C = 2\pi r$ (or) $C = \pi D$
$C = 2 \times 3.14 \times 3$ ft $C = \frac{22}{7} \times 6$ ft
$C = 6.28 \times 3$ ft.
$C = 18.84$ ft. $C = 18.85$ ft.

Practice Problems 7.17 - 7.18: Find the Circumference

17)

Ans. _____

18)

Ans. _____

b. Area of a Circle

The area of a circle is π times the radius squared, or π times the diameter squared divided by 4.

Formula: $A = \pi r^2$

$A = \pi \frac{D^2}{4}$

Example:

Find the area of a circle whose radius is 4 feet and diameter is 8 feet.

$A = \pi R^2$
$A = 3.14 \times 4 \text{ ft} \times 4 \text{ ft}$
$A = 3.14 \times 16 \text{ ft}^2$
$A = 50.24 \text{ ft}^2$

$A = \pi \frac{D^2}{4}$

$A = 3.14 \frac{(8\text{ft} \times 8 \text{ ft})}{4}$

$A = 3.14 \times \frac{64}{4}$

$A = 50.24$

Practice Problems 7.19 - 7.20: Find the area

19)

Ans. _____

20)

Ans. _____

C. VOLUMES

Volume measurement is used to measure the holding capacity of an object.

Volume is expressed in cubic units i.e. cubic inches (in^3), cubic feet (ft^3), cubic yards (yds^3), and cubic meters (m^3), gallons, etc.*

* This superscript exponent ^{3} means multiplying units x units x units and is called cubing.

For example: $m^3 = m \times m \times m$ = meter x meter x meter

The volume of a solid figure is the number of cubic units of space the figure occupies or in our case the volume of liquid it will hold.

In this section we will concern ourselves with four of the most common solids.

1. Cube
2. Rectangular Solid
3. Triangular Solid
4. Cylinder

We have two methods which we use in finding volume: The first is the old method involving the memorization of formulas, and the newer method or concept which is called the third dimensional method. Both are presented here for your consideration and use.

The concept of surface area covers the length and width dimensions. With the concept of volume, a third dimension is considered. Rather than memorizing many difficult formulas for each different shape, the concept of volume may be simplified as:

Volume = area of the surface x 3rd dimension

Area of the surface can be defined as that which gives the object its basic shape.

FIG.1

FIG.2

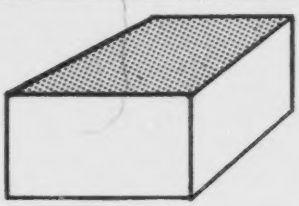


FIG. 3

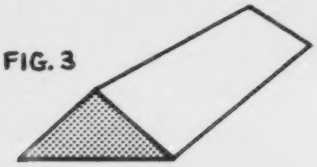


Figure 1 - In this figure the shape which defines the cylinder is the circle.

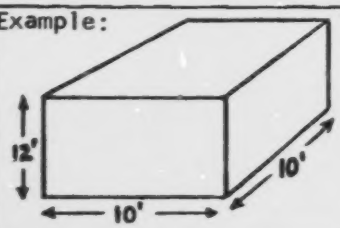
Figure 2 - In this figure the rectangle defines the shape of the object.

Figure 3 - In this figure it is the triangle which gives it its basic shape.

When we can find which surface gives an object its basic shape, finding the volume is a simple matter.

The third dimension is usually the depth or height of the figure.

Example:



Volume = area of surface x 3rd dimension (depth of rectangle
Rectangle is the basic shape
= (12) x (10) x (10)
= 120 sq. ft. x (10)
= 1200 feet^3

Example: Volume = Area of surface x third dimension-depth
the circle is the basic shape

$= (.785\ D^2) \times 12$
$= (.785)\ (10)\ (10)\ (12)$
$= 942$ cu. ft. volume

We can use this concept for calculating out the volume of all shapes.

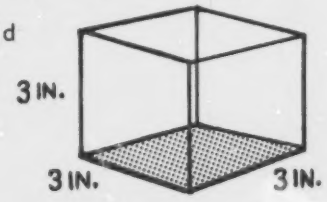
In each of the four most common solids each method will be presented when calculating the volume.

When using the third dimensional method it is not necessary to write down in the formula the basic shape of the surface area.

The basic principle of finding the cubic units in a solid is:

Find the square units of area in the figure that carries the name of the solid figure and multiply these all together.

1. Cube Solid



(Shaded area is the base)

Since the base is a square we find the area by multiplying length by width or 3 in. x 3 in. This results in 9 in^2.

To find the volume we multiply the 9 in^2 by 3 in., the height, giving us 27 in^3.

Example: The volume of a 4 inch cube is

$V = S \times S \times S$
$V = 4\ in \times 4\ in \times 4\ in$
$V = 64\ in^3$

Third Dimensional Method

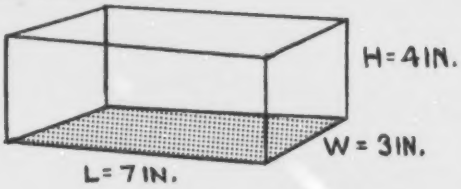
Volume = area of surface x third dimension-height
where cube is the basic shape
$= 4\ in \times 4\ in \times$ 3rd dimension
$= 16 \times 4$
$= 64\ ft^3$

Practice Problems 7.21 - 7.22: Find the volume of the following using both methods:

21) Volume of a 7 ft. cube = _____

22) Volume of a 2yd cube = _____

2. Rectangular Solid



(Shaded area is the base)

The area of the base is Length times Width which we multiply by the Height to get the Volume.

The Formula then is V = L X W X H

From the figure above V = 7 in x 3 in x 4 in
V = 84 in^3

Third Dimensional Method

Volume = area of surface x 3rd dimension
Volume = 21 x 4 (3rd dimension-height)
Volume = 84 cubic feet

Practice Problems 7.23 - 7.24

Find the volume of the following Rectangular Solids using both methods:

23)

L = 5 yds.
W = 2 yds.
H = 3 yds

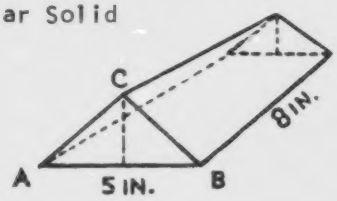
Ans. _____

24)

L = 6 ft.
W = 4 ft.
H = 3 ft.

Ans. _____

3. Triangular Solid



A B = 5 in.
H = 6 in.

The base of this solid would be the triangle formed by points a, b & c.

From a previous lesson we know the area of a triangle is the Base times the Altitude divided by 2.

The Area then is $\frac{5 \text{ in}}{2}$ x 6 in = 15 in^2

Formula: $\boxed{\text{Volume} = \frac{1}{2} \text{ base} \times \text{height} \times \text{length}}$

$V = \frac{5 \text{ in}}{2} \times 6 \text{ in} \times 8 \text{ in}$

$V = 15 \text{ in}^2 \times 8 \text{ in}$

$V = 120 \text{ in}^3$

Third Dimensional Method

Volume = Area of surface x 3rd dimension

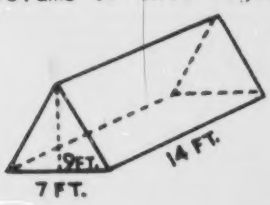
$V = 15 \text{ in}^2 \times 8 \text{ in}$

$V = 120 \text{ in}^3$

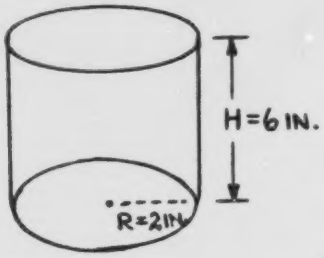
Practice Problem 7.25

Find the volume of this figure using both methods:

25)



4. Cylinder



The base of this solid would be a circle.
The r represents the radius and h represents the height.

We know that the Formula for area of a circle is πr^2

$A = \pi r^2$
$A = 3.14 \times (2\text{in})^2$
$A = 12.56 \text{ in}^2$

Formula for Volume = $\pi r^2 \times h$ of a cylinder

$V = 3.14 \times (2\text{in})^2 \times 6 \text{ in}$
$V = 75.36 \text{ in}^3$

Practice Problem 7.76

Find the volume of a cylinder using both methods when the measurements are:

r = 5 ft. h = 11 ft.

Ans. _____

DATE: _____

STUDENTS NAME: _____

CHAPTER VII ASSIGNMENT

(To be mailed to instructor)

Measure the following line segments:

Using inches

1. A _____ B

2. A _____ B

Using Centimeters

3. A _____ B

4. A _____ B

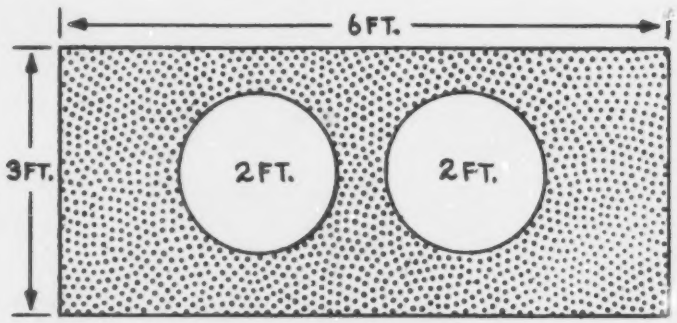
Perimeter and Area (show all calculations)

5. A rectangle is 12.8 in. by 2.2 in. Complete the area.

6. A triangle has its base = 15.8 ft., and its height = 5.5 ft. Complete the area.

7. A circle has a radius of 80 in. Find the area to the closest square inch.

8. A twin conduit has a cross-section as shown below. Complete to the nearest tenth of a sq. ft. the area of the shaded part.



9. A rectangular plot of ground 56 ft. by 40 ft. has a concrete path 6 ft. wide around its outside. Complete the area of the path.

Volume

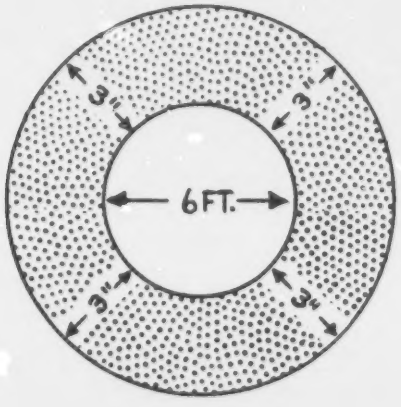
10. Find to the closest cubic foot, the total volume of a cylinder with a radius of 7 feet and height of 7 feet.

11. A reservoir is 41.5 feet long, 30.5 feet wide, and 12 feet deep. What is its capacity in cubic feet?

12. A cylindrical tank has a radius of 3 feet and a height of 8 feet. Find to the closest cubic foot its volume.

13. Find the volume of a tank measuring 10 feet wide, 15 feet long, and 6 feet deep.

14. A conduit made of concrete has a cross section as shown below. How many cubic feet of concrete are used in making 500 feet of this conduit?



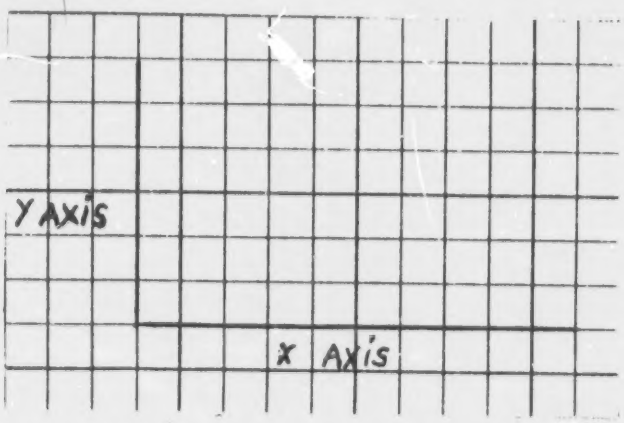
GRAPHS

A graph is a diagram or a chart which visually shows a series of changes.

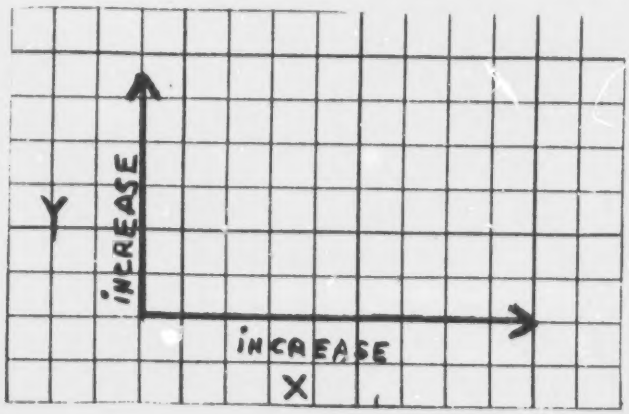
These changes may be indicated by curves, lines, dots, dashes, bars, etc. This method of representation indicates how things are changing in relation to each other.

In this case, we are going to confine ourselves to preparing and understanding line graphs. They are easy to prepare and understand. The flow charts in our treatment plants are line graphs. Charts or graphs which are drawn automatically have the advantage of making and recording continuous measurements. The graphs which we will prepare must be done manually by recording intermittent values, plotting them, and then drawing lines between these points.

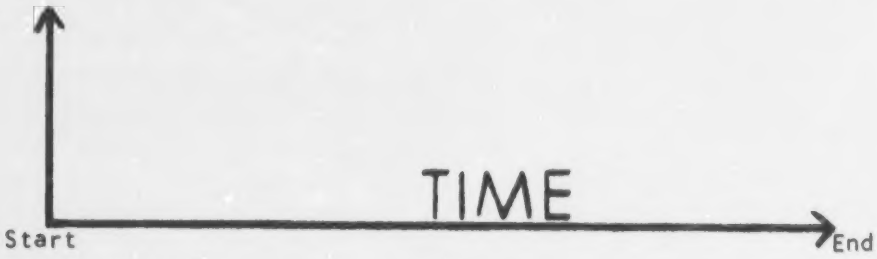
These graphs will meet two conditions. The points to be recorded will be measured in two directions, one measurement horizontally across the paper (the x axis), and the other vertically up and down the paper (y axis). The lines of the graph paper will be the same distance apart both across and up and down the paper.



The size of the numbers will increase as we move up from the point where the x and y axis meet or to the right of where the x and y axis meet.



When one of the measurements to be plotted is time, time will be plotted on the horizontal or x axis. Time will start at the point where the y axis meets the x axis and progress toward the right.

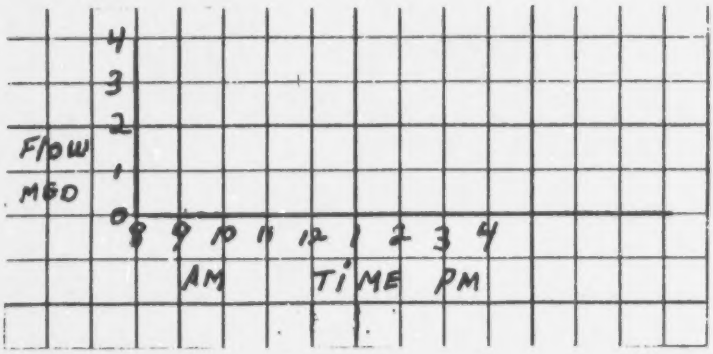


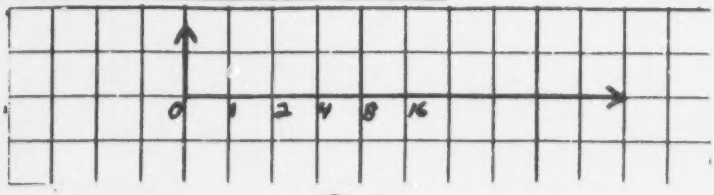
Mon	Tues	Wed	Thur	Fri	Etc.
Jan	Feb	March	April	May	Etc.
0 min	10 min	20 min		30 min	Etc.

The scales of the graph should be meaningful, accurate, and easily understood.

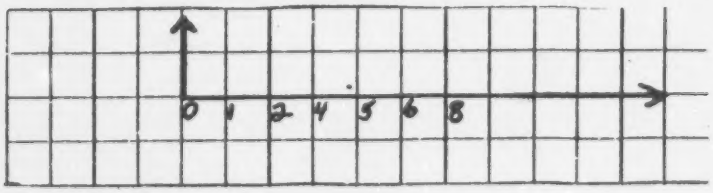
To plot flow against time, for example, we might use values such as these:

* MGD= Million Gallons per Day



DO NOT vary the units of the scales like this:

WRONG

OR

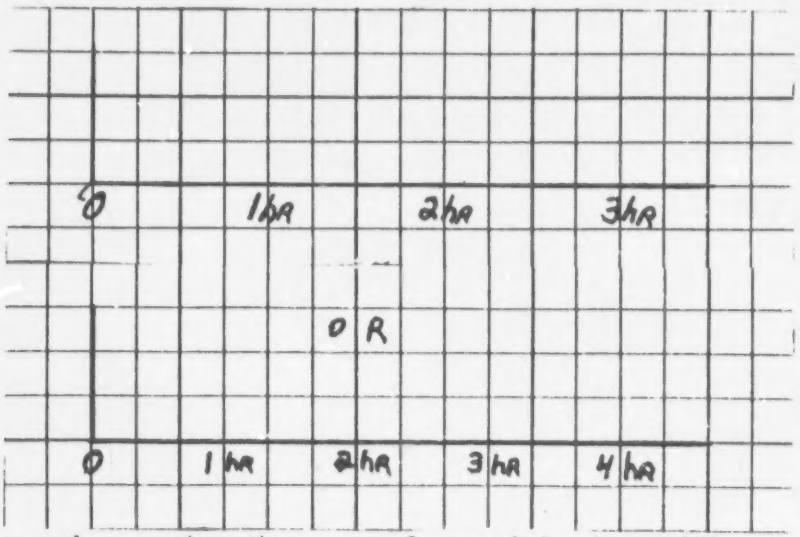


WRONG

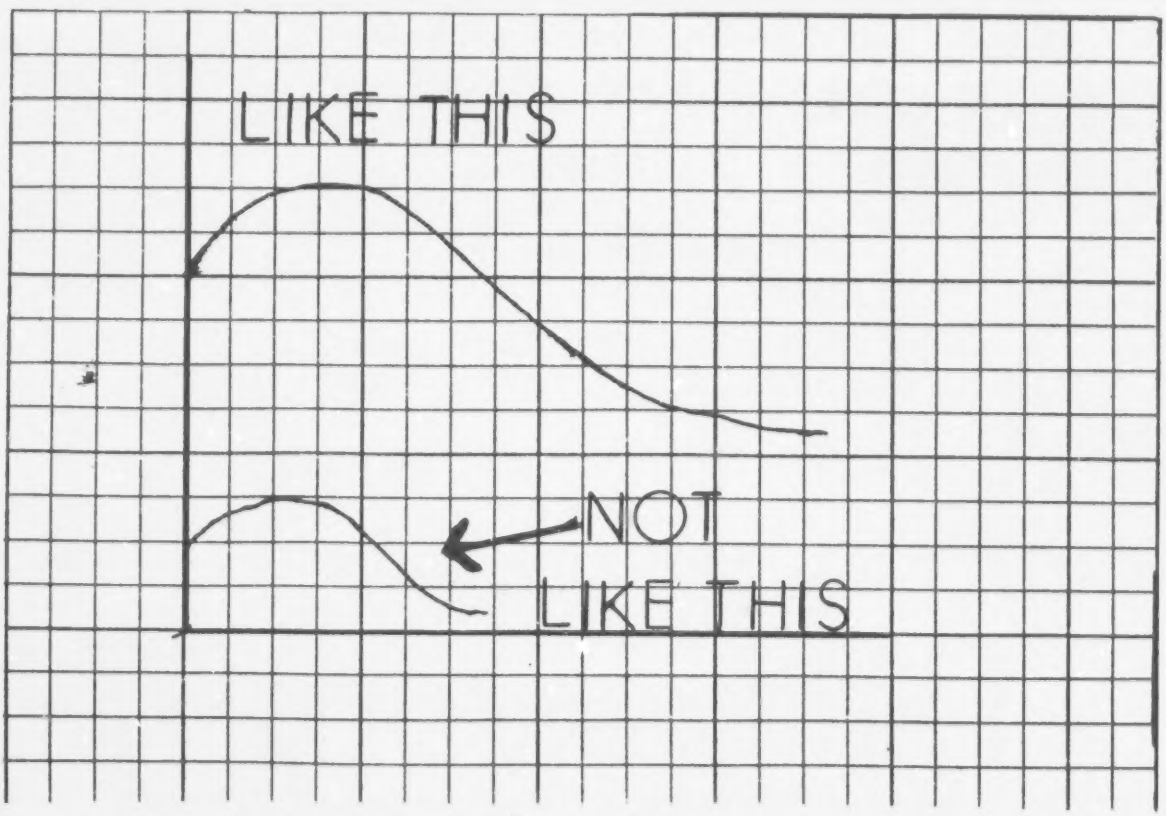
Graph Hints:

1. Make yournumbers large enough for easy interpretation.
2. Use scales which permit you to easily determine values between the recorded numbers.
3. Use scales in keeping with the numbers and values actually encountered.

For example:



Prepare your scales so that the range of recorded values will occupy the major portion of your graph.



Example

The flow at a treatment plant for the first ten days of April is as follows:

Date	Flow (M.G.D.)
1	1.0
2	1.4
3	1.5
4	1.7
5	2.0
6	2.3
7	2.5
8	1.8
9	1.5
10	1.2

We prepare our graph by using the following steps:

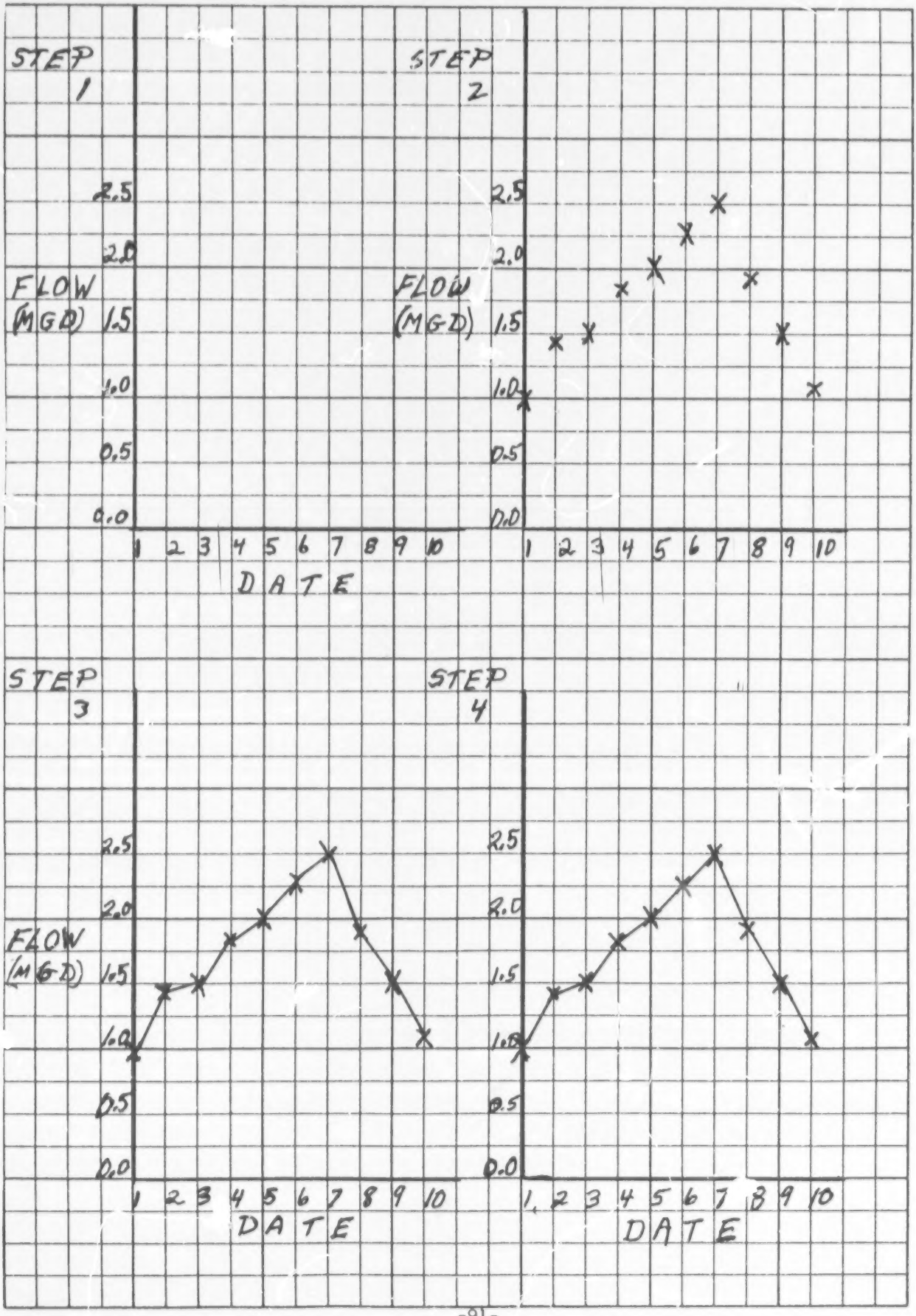
Step #1. Since time is to be plotted on the x axis, we determine the scale of values for both time and flow.

Step #2. We then plot the point from the above data.

Step #3. Next we connect the points with straight lines.

Step #4. More meaningful results may be obtained by drawing a smooth curve between the points.

Step #5. Interpret the graph from Step #4.


STEP 1
FLOW (MGD)
2.5
2.0
1.5
1.0
0.5
0.0
1 2 3 4 5 6 7 8 9 10
DATE
STEP 2
FLOW (MGD)
2.5
2.0
1.5
1.0
0.5
0.0
1 2 3 4 5 6 7 8 9 10
DATE
STEP 3
FLOW (MGD)
2.5
2.0
1.5
1.0
0.5
0.0
1 2 3 4 5 6 7 8 9 10
DATE
STEP 4
2.5
2.0
1.5
1.0
0.5
0.0
1 2 3 4 5 6 7 8 9 10
DATE

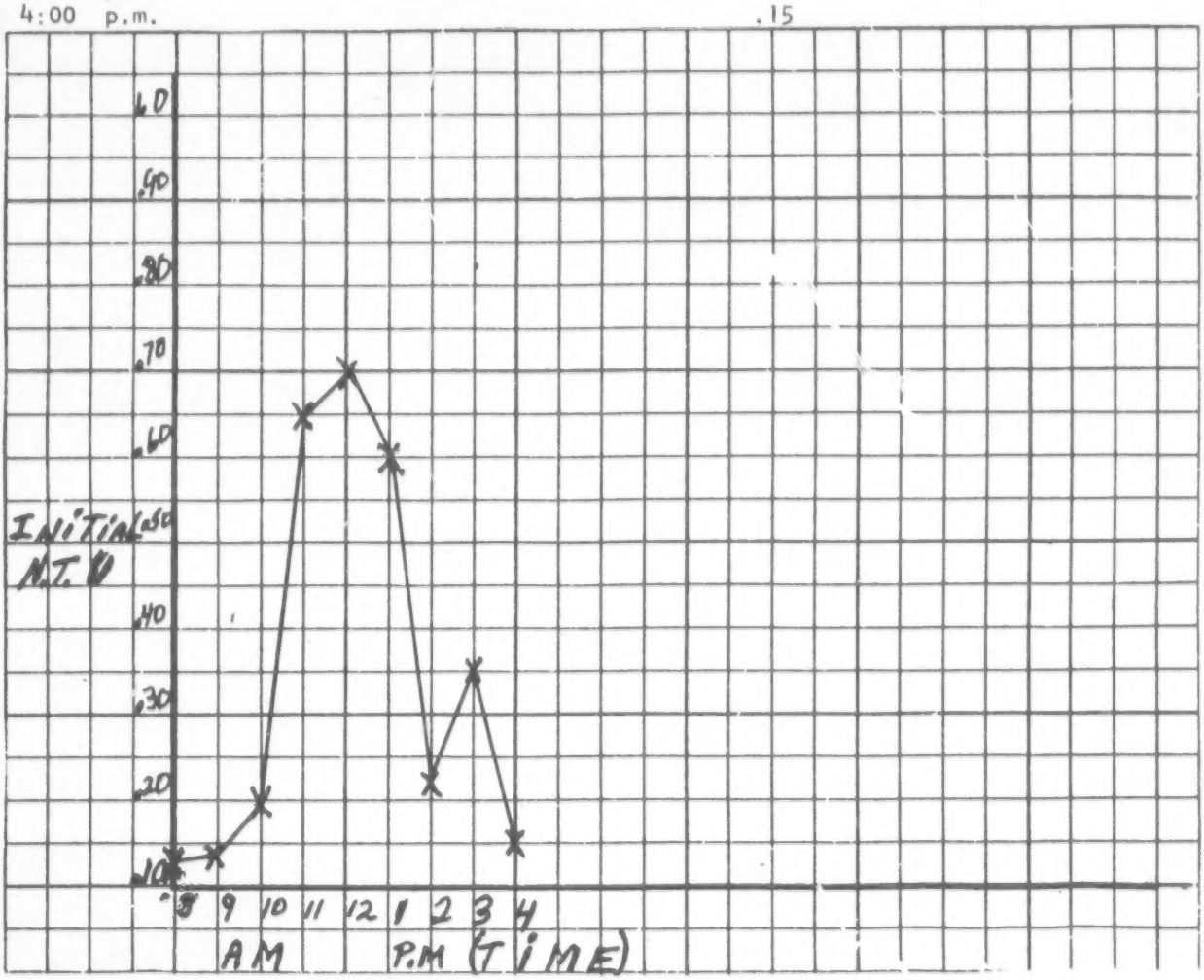
Step #5. Interpretations

(a) When did the peak flow occur? Ans. April 7
(b) What is the flow on the 3rd? Ans. 1.5 MGD
(c) How long does the peak flow last? Ans. 24 hours
(d) When does the rate of flow increase the fastest? Ans. Between 2nd to the 4th.

Example

The following turbidity readings on finished water were recorded on the 8-4 shift in June.

Time	Initial NTU
8:00 a.m.	.11
9:00 a.m.	.14
10:00 a.m.	.20
11:00 a.m.	.65
12:00 p.m.	.70
1:00 p.m.	.60
2:00 p.m.	.21
3:00 p.m.	.35
4:00 p.m.	.15



94

Date:_____

Students Name:_____

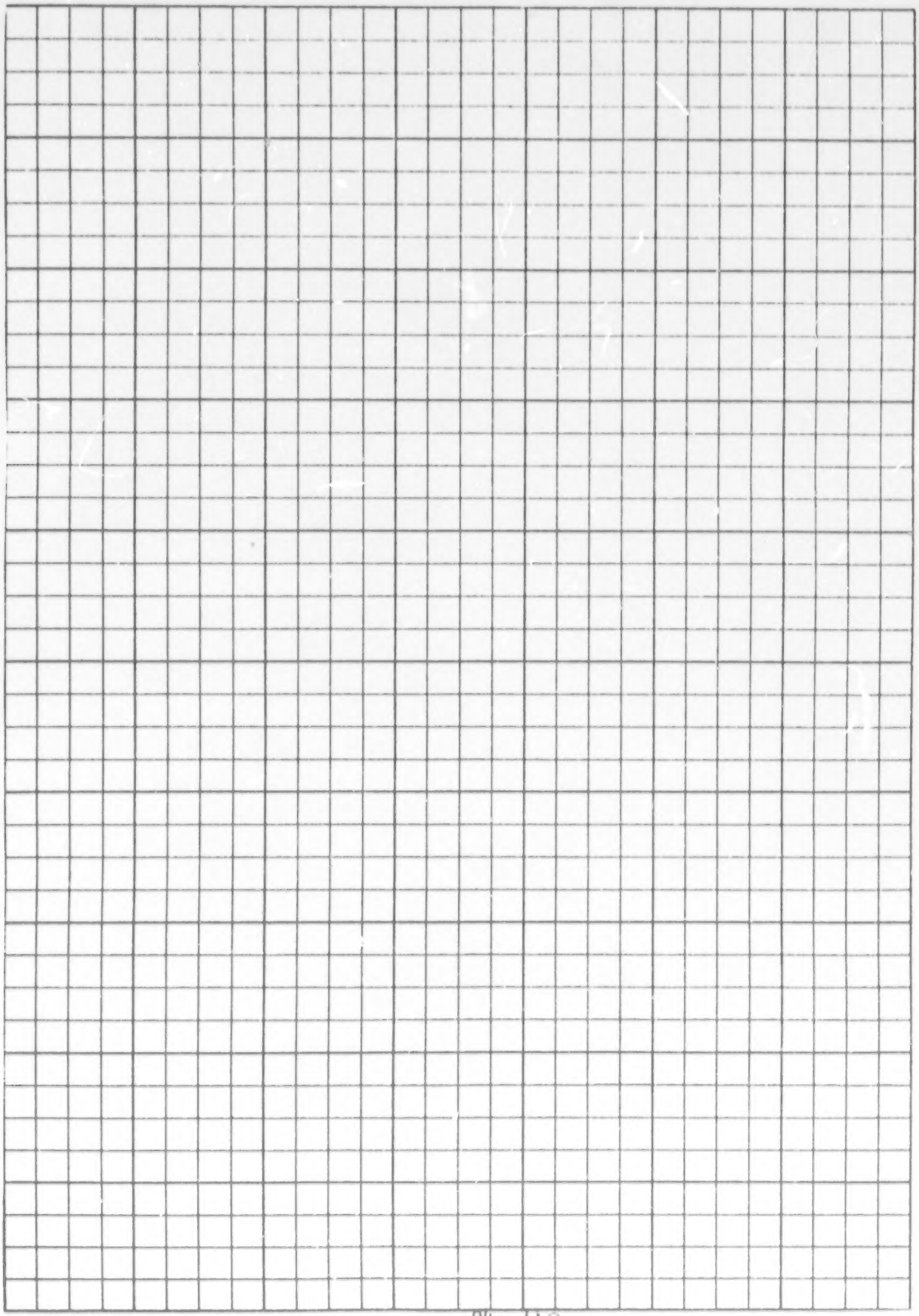
CHAPTER VIII - ASSIGNMENT SHEET

(To be returned to instructor)

Problem: Graph the following flow data:

TIME	FLOW MGD
8:00 a.m.	3.1
9:00 a.m.	3.2
10:00 a.m.	3.3
11:00 a.m.	3.7
12:00 NOON	3.8
1:00 p.m.	3.7
2:00 p.m.	3.4
3:00 p.m.	3.6
4:00 p.m.	3.7
5:00 p.m.	3.9
6:00 p.m.	4.0
7:00 p.m.	4.1
8:00 p.m.	4.4
9:00 p.m.	4.3
10:00 p.m.	3.8
11:00 p.m.	3.5
12:00 p.m. Midnight	3.2

(show work on graph on next page)

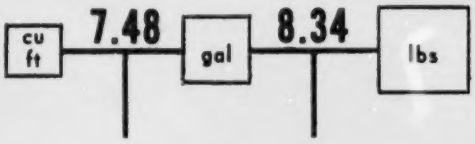


SECTION I CHAPTER IX

CONVERSIONS

A. VOLUMES TO WEIGHTS

We can best express the relationships among cubic feet, gallons, and pounds for water by a diagram.



*lbs. = pounds

1 cu. ft. = 7.48 gallons 1 gallon = 8.34 pounds

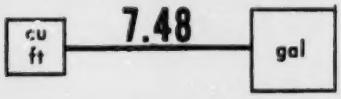
The lines which connect the boxes can be thought of as bridges which tie togehter any two boxes. Depending on the size of the boxes, we can determine whether multiplication or division is the appropriate mathematical process.

Multiplication - It is associated with <u>increasing</u> a number - we use this function when moving from a <u>smaller</u> box to a <u>larger</u> box.

Division - It is associated with <u>decreasing</u> a number - we use division when moving from a <u>larger</u> box to a <u>smaller</u> box.

Example

A tank has a capacity of 60,000 cubic feet. What is the gallon capacity of the tank?

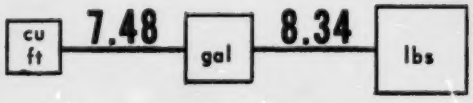


When we are converting from cubic feet to gallons, we are moving from a smaller box to a larger box. Therefore, multiplication by 7.48 is used.

(60,000) x (7.48) = 448,800 gal. Vol.

Example

500,000 pounds of alum sludge is sent to the drying bed. Assuming that each gallon of sludge weighs 8.34 lbs. How many cubic feet of sludge will be sent to the drying bed?



Since we are converting from lbs. to cubic feet, we are moving from a larger number (box) to a smaller one so we use division.

$$\frac{500{,}000 \text{ lbs.}}{8.34 \text{ lbs/gal.}} = \frac{59{,}952 \text{ gal.}}{7.48 \text{ gal/cu.ft.}} = 8{,}015 \text{ cubic feet}$$

Practice Problems 9.1 - 9.3

Cubic feet to Pounds

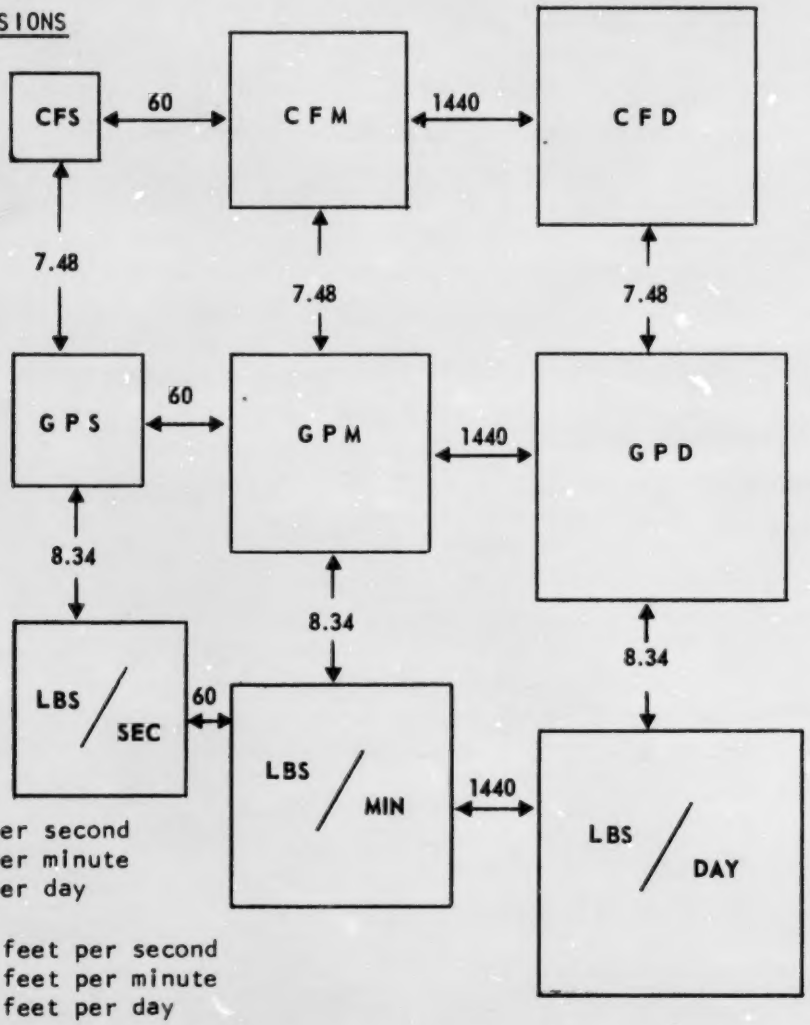
Using the information discussed previously do the following practice problems.

1. Change 1000 cubic feet of water to gallons _____ ans.

2. What is the weight of three cubic feet of water? _____ ans.

3. The net weight of a tank of water is 750 lbs.
How many gallons does it contain? _____ ans.

B. FLOW CONVERSIONS



gps = gallons per second
gpm = gallons per minute
gpd = gallons per day

C.F.S. = cubic feet per second
C.F.M. = cubic feet per minute
C.F.D. = cubic feet per day

Points to Remember About the Previous Diagram

1. The lines connecting the boxes are bridges.
2. The boxes are not drawn to scale, but to help you in deciding whether multiplication or division is appropriate.

*Remember:

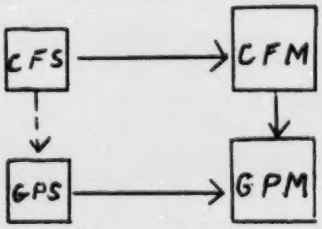
Smaller ⟶ Larger Multiplication is called for
Larger ⟶ Smaller Division is called for

3. The relationship of the above chart should be understood NOT MEMORIZED.

4. In going from CFS to CFM this is merely a time frame associated with cubic feet. In going from CFS to gps, we are changing from cubic feet to gallons or vice versa.

Example

If the water flow in a main is 2.3 CFS, what is this flow expressed at gpm? (assume the flow is steady and continuous)

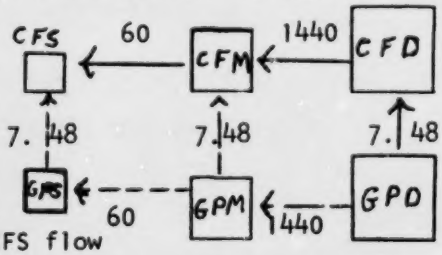


(2.3) (60) (7.48) = 1032 gpm flow

We can go two possible ways from CFS to gpm. Note that in each there are two factors 60 and 7.48. In each case we are going from smaller to larger, so we multiply

Example

The flow to a distribution reservoir is 2,450,000 gpd at this rate, what is the average CFS flow?



We can go 3 possible routes from gpd to CFS. In all three we are going from a larger box to a smaller one. Hence, we divide.

$$\frac{2{,}450{,}000}{(7.48)\ (1440)\ (60)} = 3.79 \approx 3.8 \text{ CFS flow}$$

to CFD to CFM to CFS

99

Practice Problems 9.4 - 9.5

4) The flow to a clearwell storage tank is 1,500,000 gpd. What is the flow in CFS?

ANS. _____

5) The water flow from a well is 450 gpm. What is the flow in gps.

ANS. _____

C. CONVERSIONS WITHIN THE METRIC SYSTEM

Before we can perform any conversions, it is absolutely essential that we are at the very least acquanited with the six common metric prefixes, which are required to do these conversions.

THESE PREFIXES SHOULD BE LEARNED BEFORE ANY CONVERSIONS ARE ATTEMPTED.

Primary Unit

Kilo Hecto Deca Meter Deci Centi Milli

Examples include

meters	linear measurement
liters	capacity measurement
grams	weight measurement

Metric System is based on units (powers) of TEN.

In conversions of linear measurement, capacity measruement, and weight measurement, EACH PLACE VALUE will represent ONE DECIMAL MOVE. This system of conversions is best explained by example.

Example

Convert 1 meter to decimeters

_____ Deci

1 Meter _____ ↑ We move 1 place to the right

Example

Convert 1 gram to (a) Decigrams, (b) Centigrams, (c) milligrams

_____ Deci Centi Milli

1 Gram _____ ↑ ↑ ↑

(a) 1 place to the right

1.0 grams = 10 decigrams

(b) 2 placed to the right

1.00 grams = 100 centigrams

(c) 3 places to the right

1.000 grams = 1000 milligrams

This system of conversion will apply whether converting from primary units, as in examples 1 and 2, or to any other unit.

Example Convert 1 decimeter to millimeters

Primary Unit	Deci	Centi	Milli
1 Decimeter			↑

2 places to the right

1.00 decimeters = 100 millilliters

Example Convert 3.5 kilograms to grams

Kilo	Hecto	Deca	Primary Unit
3.5 Kilogram			↑

3 places to the right

3.500 kilograms = 3500 grams

In the following examples we will convert from by moving "x" number of places to the left.

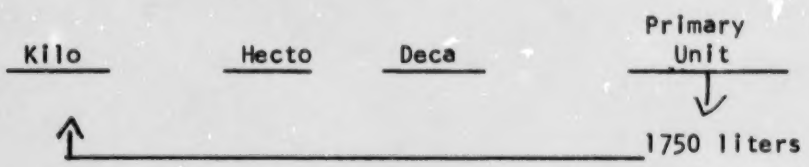
Example Convert 0.28 centimeters to meters

Primary Unit	Deci	Centi	Milli
↑		0.28 centimeters	

2 places to the left

00.28 centimeters = .0028 meters

Example: Convert 1750 liters to kiloliters



3 places to the left

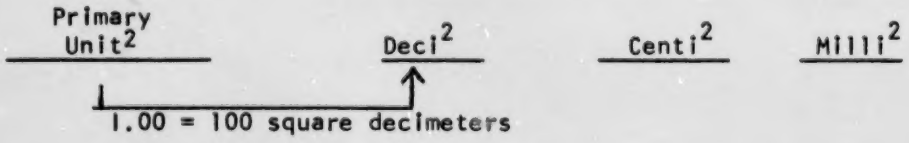
1750. = 1.75 kiloliters

When moving the decimal point to the left, be careful to move the decimal from its present position, being sure to count every number to the left as a decimal move.

One final conversion we should be aware of is converting to square terms or cubic terms.

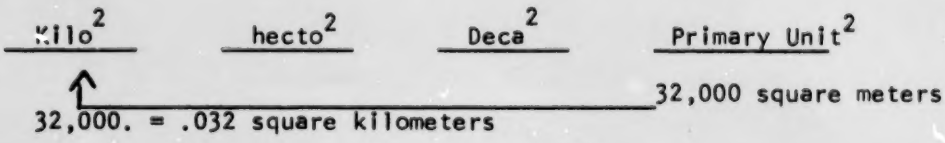
In converting to square units, the times squared indicates the exponent of 2 which means each place value move will be worth 2 decimal place moves. One final note; square and cubic conversions will focus on meters and related terms; since weight and capacity are not expressed in square or cubic units.

Example: Convert square meter to square decimeter



Remember: In square terms, each place move is worth 2 decimal place moves.

Example: Convert 32,000 square meters to square kilometers



3 place moves to the left indicates 6 decimal moves.

D. ENGLISH SYSTEM TO METRIC SYSTEM

Conversion Identities

Metric to English	English to Metric
Linear Measurement	
1 centimeter = 0.3937 inches	1 inch = 2.540 cm
1 meter = 3.281 feet	1 foot = 0.3048 m
1 meter = 1.0936 yards	1 yard = 0.9144 m
1 kilometer = 0.6214 miles	1 mile = 1.609 km
Square Measure	
1 sq. centimeter = 0.155 sq. in.	1 sq. in. = 6.4516 cm
1 sq. meter = 10.76 sq. ft.	1 sq. ft. = 0.0929 m
1 sq. meter = 1.196 sq. yd.	1 sq. yd. = 0.8361 sq. m
Cubic Measure	
1 cu. centimeter = 0.061 cu. in.	1 cu. inch = 16.39 cu. cm
1 cu. meter = 35.3 cu. ft.	1 cu. foot = 0.0283 cu. m
1 cu. meter = 1.308	1 cu. yard = 0.7645 cu. m
Capacity	
1 liter = 61.025 cu. in.	1 cu. inch = 0.0164 liters
1 liter = 0.0353 cu. ft.	1 cu. foot = 28.32 liters
1 liter = 0.2642 gal (U.S.)	1 gal. U.S. = 3.785 liters
Weight	
1 gram = 15.43 grains	1 grain = 0.0648 g.
1 gram = 0.0353 ounces	1 ounce = 28.35 g.
1 kilogram= 2.205 pounds	1 pound = 453.6 g.
	1 pound = 7000 grains

APPLICATION OF ENGLISH TO METRIC CONVERSIONS

English System to Metric System

1. Conversions within the Metric system are much easier to perform because we are merely moving decimal places.

2. In the examples which follow, if we have a choice between making a change within the English system or within the Metric system, we will choose to make the change within the Metric system.

Example Convert 2 feet to meters

1 foot = 0.3048 meters (from coversion chart on page 100)

We calculate this by multiplying by 2 on each side of the equation.

(2) (1 foot) = (0.3048) (2)

2 feet = 0.6096 meters

Example Convert 3.5 liters to cubic feet

1 liter = 0.0353 cubic feet (from conversion chart on page 100)

So our calculation is to multiply each side of the equation by 3.5.

(3.5) (1 liter) = (0.0353) (3.5)

3.5 liters = 0.1236 cubic feet

Example

The cross-sectional area of a channel is 8.5 square feet. How many square meters is this? Remember from our chart: 1 sq. ft. = 0.0929 square meters. So our calculation is to multiply each side of the equation by 8.5.

(8.5) (1 sq. ft.) = (0.0929) (8.5)
8.5 sq. ft. = 0.7897 sq. meters

These cross system conversions are not difficult when we are able to find an identity which directly relates the two terms in the statement of the problem. Identity means something which is equal to the original. Such as 1 inch = 2.540 cm.

When we are not able to find an identity which directly relates the terms stated in the problems, we may wish to make the solution of the identity so that the ENGLISH TERM OF THE PROBLEM AND THE ENGLISH TERM OF THE IDENTITY CORRESPOND. If we do this, necessary changes of the problem statement or identity will be made on the metric side which is an easier change to make. This is particularly true for conversions within the square or cubic measurements.

The following examples will illustrate this concept.

Example

The water depth in a channel is 1.4 feet. How many centimeters is this?

From our chart we find that there is no relationship between feet and centimeters. We do find, however, that there is a feet to meters and a inches to centimeters relationship.

1 inch = 2.540 centimeters
1 foot = 0.3048 meters

So we have two possible solutions:

Solution 1

Using the identity:

1 inch = 2.540 cm
1.4 feet = 16.8 inches

So we restate our problem:

The water depth is 16.8 inches. How many centimeters is this?

1 in = 2.540 cm

(16.8) (1) = (2.540) (16.8)

16.8 = 42.67

Solution 2

Using the identity:

1 foot = 0.3048 meters

First we have to convert over meters to centimeters by moving the decimal 2 places to the right. Therefore, we have:

1 foot = 30.48 cm

(1.4) (1) = (30.48) (1.4)

1.4 = 42.67 cm

Example

If we are using 250 grams of a chemical for a particular solution, How many pounds of the chemical are we using?

We are making a Metric to English conversion. If we refer to the conversion table, there is nc direct relationship of grams to pounds. We do find the following however:

1 gram = 0.0353 ounces
1 kilogram = 2.505 pounds

We will, of course, prefer the second identity of pounds. We will, however, do both solutions so that a comparison can be made.

Using the identity:

1 gram = 0.0353 ounces

In this case we can calculate to ounces and convert to pounds.

1 gram = 0.0353 ounces

(250) (1) = (0.0353) (250)

250 = 8.825 ounces

16 ounces = 1 pound

8.825 divided by 16 ounces = ? lbs..

$\frac{8.825}{16}$ = .55 lbs.

Using the identity:

1 kilogram = 2.205 pounds

First we change 250 grams to kilograms by moving the decimal 3 places to the left.

.250. = .250 kilograms

Therefore:

1 kg. = 2.205 pounds

(.250) (1) = (2.205) (.250)

0.25 = .55 pounds

E. MILLIGRAM/LITER TO POUNDS

Milligrams per liter is a measure of concentration used frequently in treatment plant calculations.

Mathematically, the concentration may be expressed as:

$$\frac{\text{Mg solids}}{\text{1 liter}}$$

Another method of reporting solids concentration is in terms of parts per million (ppm). For most purposes, mg/l concentration and ppm concentration may be used interchangeably. That is:

Mg/l = ppm

By definition ppm means the number of parts of any characteristic per 1,000,000 parts of that characteristic.

5 ppm of solids is 5 lbs. of solids per 1,000,000 pounds of solution.

5 mg/l of solids = 5 lbs. of solids per 1,000,000 pounds of solution.

When making the generalization from ppm, you must exercise caution that the terms used are not mixed. Preferably both terms should be expressed in pounds.

For Instance:

4 ppm solids CANNOT BE INTERPRETED AS 4 pounds of solids per 1 million gallons of flow.*

*In this case, pounds and gallons are mixed terms.

For illustration lets consider converting from a mg/l concentration to pounds in an actual water supply situation.

Example

If the chlorine residual for an average day is 0.5 mg/l, how many lbs/day of chlorine are used when the average flow is 10.3 million lbs/day.

Step 1: First convert .5 mg/l CL_2 to pounds of chlorine per 1 million pounds of flow (solution).

$$.5 \text{ mg/l } CL_2 = \frac{.5 \text{ lbs } CL_2}{1{,}000{,}000 \text{ lbs flow}}$$

Step 2: Multiply the pounds of CL_2 per 1,000,000 lbs. of flow by the flow per day.

$$\frac{.5 \text{ lbs } CL_2}{1{,}000{,}000 \text{ lbs. flow}} \times \frac{10{,}300{,}000 \text{ lbs. flow}}{\text{day}}$$

Step 3: Cancel terms

$$\frac{.5 \text{ lbs } CL_2}{1{,}000{,}000 \text{ lbs. flow}} \times \frac{10{,}300{,}000 \text{ lbs. flow}}{\text{day}}$$

Step 4: Complete multiplication

$$\frac{.5 \text{ lbs } CL_2}{10} \times \frac{103}{\text{day}} = \frac{5.15 \text{ lbs. } CL_2}{\text{day}} \quad \text{(answer)}$$

Normally the daily plant flow is expressed as million gallons per day (m.g.d.). We will convert mgd to million lbs/day flow by multiplying by 8.34, since each gallon of water weighs 8.34 lbs.

(MGD) (8.34) = million lbs/day flow.

Example

To add 2.5 mg/l to a water which is flowing through a plant at the rate of 1 mgd, what setting should be made on the chlorinator in pounds per day?

Step 1: Convert 2.5 mg/l CL_2 to pounds of CL_2 to be added per 1,000,000 lbs. of flow per day.

$$2.5 \text{ mg/l } CL2 = \frac{2.5 \text{ lbs. } CL_2}{1{,}000{,}000 \text{ lbs. flow}}$$

Step 2: Convert 1 mgd flow to pounds per day of flow.

$$\frac{1{,}000{,}000 \text{ gallons}}{\text{day}} \times \frac{8.34 \text{ lbs.}^*}{\text{gallons}} = \frac{8{,}340{,}000 \text{ lbs. flow}}{\text{day}}$$

(*refer to conversion factor above for water)

Step 3: Multiply lbs. of flow per day by lbs. CL_2 per pounds of flow.

$$\frac{8{,}340{,}000 \text{ lbs. flow}}{\text{day}} \times \frac{2.5 \text{ lbs. } CL_2}{1{,}000{,}000 \text{ lbs. flow}} =$$

$$\frac{834}{\text{day}} \times \frac{2.5 \text{ lbs. } CL_2}{100} = \frac{20.85 \text{ lbs. } CL_2}{\text{day}} \quad \text{(answer)}$$

Practice Problems 9.6 - 9.7

6) If you were required to add 0.6 mg/l of flouride to your water supply, How many pounds a day of sodium flouride would you use when the average flow is 1,800,000 million lbs/day?

ans. _____

7) 2.5 mg/l of chlorine are added to a water supply. How many pounds a day would be used for a flow of 8.3 millions lbs/day?

ans. _____

F. MILLIGRAMS PER LITER TO PERCENTAGE

When we first began our discussion of milligrams per liter, it was pointed out.

mg/l = ppm = milligrams/1,000,000 milligrams.

This concept is the key to determining the percentage solids content.

Example: The alum sludge concentration of the settling basin is 340 mg/l. What is the percent solids concentration of this water?

In general: $\text{Milligrams/Liter} = \frac{\text{Milligrams}}{1{,}000{,}000 \text{ milligrams}}$

In this Example: 340 milligrams/liter = 340 milligrams/1,000,000 milligrams.
We can say this because-(1 liter = 1,000,000 milligrams)

In converting to percentage we wish to know how many milligrams there are per 100 milligrams. To convert mg/l to percent move the decimal point 4 places or numbers to the left, in both the numerator and the denominator.

$$\frac{340 \text{ milligrams}}{\text{Liter}} = \frac{.0340. \text{ Milligrams}}{1{,}000{,}000 \text{ Milligrams}}$$

$$= \frac{.034 \text{ milligrams}}{100 \text{ milligrams}}$$

$$= .034\%$$

G. MILLIGRAMS PER LITER TO GRAINS PER GALLON

1 grain per gallon = 1,000,000 grains per million gallons.
Since 1 lb contains 7,000 grains
Then 1,000,000 ÷ 7,000 = 143 lbs.
Therefore: 143 lbs. per 1 million gallons = 1 grain per gallon

We can sum all this up into a general useful formula:

$$\text{Grains Per Gallon} = \frac{\text{Lbs. of Chemical Used}}{\text{Water Flow in M.G.D.} \times 143}$$

Example: A plant uses 825 pounds of Alum to treat a raw water flow of 7.2 m.g.d. How many grains per gallons are used?

$$\text{Grains per Gallon} = \frac{(825 \text{ lbs.}) \text{ lbs. of chemical used}}{\text{water flow in M.G.D.} \times 143 \ (7.2 \text{ mgd})}$$

$$= \frac{825}{7.2 \times 143}$$

$$= 0.8 \text{ g.p.g.}^*$$

*g.p.g. = Grains Per Gallon

Example: A plant with a raw water flow rate of 2.5 million gallons a day, has a desired alum treatment of 1.8 gpg. How many pounds of alum are needed for the desired alum treatment?

$$\text{Grains Per Gallon} \ (1.8) = \frac{(x) \text{ lbs. of chemical used}}{\text{water flow in MGD} \times 143 \ (2.5)}$$

$$x = 1.8 \times 2.5 \times 143$$

$$x = 643 \text{ lbs.}$$

Practice Problems 9.8 - 9.9

8) 1400 pounds of alum are used per day with a flow of 7.6 mgd. What is the average dose in gpg?

ans._____

9) How many pounds of lime must be added to 20,000 gallons to give a concentration of 5 gpg?

ans._____

H. PARTS PER MILLION TO MILLIGRAMS PER LITER

PARTS PER MILLION (ppm) means the number of parts in a million parts.

Milligrams per liter means nearly the same thing. We therefore say, ppm = mg/l and vice versa.

These parts may be grams, ounces, tons, gallons, or any other unit.

Since ppm or mg/l is usually expressed in units of weight, whenever we are dealing with any other unit, such as gallons or cubic feet, we must convert to weight before we can solve the problem.

Example: How much is one ppm or mg/l in a million gallons of water?

One gallon of water weighs 8.34 pounds, therefore on million (1,000,000) gallons will weigh?

1,000,000 x 8.34 = 8,340,000 pounds

(Answer) $\frac{8,340,000}{1,000,000}$ = 8.34 lbs. to equal 1 ppm in 1,000,000 gallons of water

Example: How much is one ppm or mg/l in 15,000 cubic feet of water?

One cubic foot of water weighs 62.4 pounds

15,000 x 62.4 = 936,000 lbs.

(Answer) $\frac{936,000}{1,000,000}$ = 0.936 lbs. (to equal 1 ppm in 15,000 cu. ft. of water).

Example:

Given: Raw water flow rate = 7.2 M.G.D.
lbs. of alum used = 825

Find the alum dosage in P.P.M. or Mg/l?

$$\text{P.P.M. or Mg/l} = \frac{\text{lbs. of chemical used}}{\text{water flow (MGD)} \times 8.34}$$

$$\text{P.P.M. or Mg/l} = \frac{825}{7.2 \times 8.34}$$

$$= \frac{825}{60}$$

= 13.7 mg/l or ppm

DATE:_____

STUDENTS NAME:_____

Chapter IX - Conversions - Assignment Sheet

(To be returned to instructor)

1. Find the volume of a water basin 15 feet long, 6 feet wide, 4.5 feet deep?

2. A pump delivers 25 g.p.m. of water. How much time is required to fill a 1,000 gallon tank?

3. During ten hours per day, a station pumps with a 500 g.p.m. unit. What is the plant out put in M.G.D. per 10 hours?

4. Change the following:

   A. 800 grams to kilograms _____.
   B. 14 liters to cubic centimeters_____.
   C. 400 cubic feet of water weighs _____lbs.
   D. 1 gallon of water weighs _____grams.
   E. 250 gallons of water weighs_____pounds.
   F. 700 pounds of water occupy _____gal.
   G. 75 liters of water weighs _____lb.

5. Change 5 g.p.g. to p.p.m.

6. Change 15 grams per 400 gallons to p.p.m.

7. Change 3 pounds per 2000 gallons to p.p.m.

8. A 10 gallon sample of water contains 6 grams of solids.

   A. Compute the pound per 1000 gallons.

   B. Compute the g.p.g.

9. If 220 pounds of alum are added to 700,000 gallons of water, what is the dosage in g.p.g.?

(OVER) 111

10. Change 3 g.p.g. to p.p.m. or mg/l.

11. How many pounds of lime must be added daily to a flow of 1 million gallons per day (m.g.d.) to give a dosage of 10 mg/l?

12. A tank contains 10,000 gallons of water and 15 pounds of solids. What is the concentration of solids in p.p.m. or mg/l?

13. How many pounds of lime must be added to 20,000 gallons to give a concentration of 5 g.p.g.?

14. Calculate the pounds of solids entering a tank each day if the flow is 4 m.g.d. and the concentration of solids is 200 p.p.m.

15. There are 7,000 pounds of solids in 2 million gallons. Calculate the mg/l (p.p.m.)?

SECTION II PROCESS CALCULATIONS

SECTION II CHAPTER I

CHLORINATION

A. CALCULATING FEED RATE:

Calculating the chlorine feed rate or any chemical feed rate is a simple mathematical procedure. To calculate chemical feed rate use the following formula:

$$\text{Chlorine Feed Rate lbs/day} = (\text{dose mg/l})\ (\text{flow m.g.d.})\ (8.34\ \text{lbs/gal})$$

Example 1:

How many pounds of chlorine a day will be used if the dosage is 1.2 mg/l for a flow of 1,600,000 m.g.d.?

Chlorine Feed Rate lbs/day = (dose mg/l) (flow mgd) (8.34 lbs/day)

Plugging in the numbers given:

$$= (\text{dosage } 1.2\ \text{mg/l})\ (\text{flow } 1.6)\ (8.34\ \text{lbs/day})$$

We have the following equation to solve

$$= (1.2)\ (1.6)\ (8.34)$$
$$= \underline{16.0\ \text{lbs/day}}$$

Example 2:

How many pounds of CL_2 a day will be used if the dosage is 0.6 mg/l for a flow of 600,000 g.p.d.

$$\text{Chlorine Feed Rate} = (\text{dose mg/l})(\text{flow m.g.d})(8.34\ \text{lbs/day})$$
$$= (\text{dose } .6\ \text{mg/l})\ (\text{Flow } .6\ \text{mgd})(8.34\ \text{lbs/day})$$
$$= (0.6)(0.6)(8.34)$$
$$= 3.00\ \text{lbs/day}$$

Practice Problems 1.1 - 1.2

1) How many pounds of chlorine will be used if the dosage is 0.4 mg/l for a flow of 5,300,000 g.p.d.

Ans. _____

2) How many pounds of chlorine will be used if the dosage is 0.2 mg/l for a flow of 1 m.g.d.

Ans._____

B. CHLORINE DEMAND

When calculating chlorine demand we can use the following formula:

Chlorine Demand mg/l = Chlorine Dose mg/l - Chlorine Residual mg/l

The following example will illustrate how this formula is used.

Example 3:

Chlorine Demand mg/l = Chlorine Dose mg/l - Chlorine Residual mg/l

= 1.0 mg/l - 0.5 mg/l

= 0.5 mg/l

Practice Problems 1.3 - 1.5

What is the chlorine demand in mg/l for the following:

	Dose mg/l	Chlorine Residual mg/l	
3)	1.6	0.4	Ans._____
4)	0.3	0.2	Ans._____
5)	1.0	0.4	Ans._____

C. CHLORINE RESIDUAL

In calculating the chlorine residual, we use the same formula as we did for finding chlorine demand.

Chlorine Demand mg/l = Chlorine Dose mg/l + Chlorine Residual

The only difference in using this formula is that we will have to subtract the chlorine dosage from the chlorine demand to find the residual.

The following examples should help to illustrate this.

Example 4:

If the chlorine dosage is 6 mg/l, what must the chlorine residual be if the chlorine demand is 3.3 mg/l?

Chlorine Dosage mg/l = Chlorine Demand mg/l + chlorine residual mg/l.

3.3 mg/l = 6.0 mg/l — x mg/l

x mg/l = 6.0 mg/l - 3.3 mg/l

Chlorine Residual = 2.7 mg/l

Before we can calculate the chlorine residual, we have to express the dosage information as mg/l. Because chlorine dosage, demand and residual are expressed in mg/l or ppm. (Remember ppm = mg/l).

$$\text{Mg/l or PPM} = \frac{\text{lbs of chemical used}}{\text{flow (m.g.d.)} \times 8.34}$$

Example 5:

A water treatment plant has a daily flow of 3.1 mgd. If the chlorinator setting is 220 lbs/day, and the chlorine demand is 3.6 mg/l, what is the chlorine residual?

$$\text{mg/l} = \frac{220 \text{ lbs/day } Cl_2}{3.1 \times 8.34} = 8.5 \text{ mg/l dosage}$$

Dosage Mg/l = demand mg/l + residual mg/l

7.5 mg/l = 6.9 mg/l + x mg/l

x = 7.8 - 6.9

Chlorine Residual = 0.9

Practice Problems 1.6 - 1.7

Find the chlorine residual in the following:

6. If the chlorine dosage in 10 mg/l, what must the chlorine residual be if the demand is 2.6 mg/l?

7. A treatment plant has a demand of 0.4, and the dosage is 1.2 mg/l. What is the residual?

D. HYPOCHLORITE CALCULATIONS

A hypochlorite is a compound of chlorine and another chemical which is generally applied in solution form.

The chlorine which is found in these hypochlorite compounds is known as "available chlorine".

The fundamental concept to be kept in mind is that once we have determined how many pounds of chlorine will be required for disinfection (dosage) we will always need more pounds of hypochlorite.

For example, suppose we calculate that 100 lbs/day of chlorine will be the required dosage for a particular water. It may be necessary to dose 130 to 300 lbs/day of hypochlorite, depending on the amount of available chlorine in the compound.

Example 6:

How many pounds of available chlorine are in 45 pounds of 70% HTH?

The relationship between the available chlorine and the hypochlorite may be restated as:

Verbal: 70% of the HTH is available chlorine.

Calculation: (70%) (hypochlorite) = available chlorine.

(.70) (45 lbs/day HTH) = x lbs/day available Cl_2

Solving for x = (70%) (45 lbs/day)

x = 31.5 lbs/day of available Cl_2

Example 7:

A water requires a dosage of 120 lbs/day of chlorine for disinfection. If we are using a hypochlorite with 28% available chlorine, how many lbs/day of the hypochlorite will be required?

Again the mathematical calculation will be more obvious if we state the problem in a slightly different way.

Verbal: 28% of the hypochlorite is available chlorine.

Calculations: (28%) (hypochlorite) = available chlorine

(.28) (x lbs/day hypochlorite) = 120 lbs/day chlorine

Solving for "x": $x \text{ lbs/day} = \frac{(120 \text{ lbs/day available chlorine})}{.28}$

$$x = \frac{120}{.28}$$

x = 429 lbs/day hypochlorite

At times we will have to calculate the lbs/day chlorine dosage requirement BEFORE making any calculations regarding hypochlorite dosage. The following example will illustrate to you both concepts.

Example 8:

A test of the effluent in the clear well shows that the required dosage of chlorine is 0.6 mg/l. The average daily flow at the treatment plant is 1 mgd. If we will be using a hypochlorite solution with 68% available chlorine, how many lbs/day hypochlorite will be required?

First, in accordance with our statement above we must determine the required chlorine dosage. We must merely convert the mg/l or p.p.m. dosage to lbs/day.

$$0.6 \text{ mg/l } Cl_2 \times \frac{0.6 \text{ lbs } Cl_2}{1 \text{ mil lbs. flow}} \times (1) \text{ MGD} \times (8.34 \text{ lbs/gal})$$

$$= 5.00 \text{ lbs/day chlorine dosage}$$

If we require 5.00 lbs/day of chlorine, we will require more pounds of hypochlorite which is not 100% chlorine.

Verbal: 68% of the hypochlorite is available chlorine.

Calculation: (68%)(hypochlorite) = available chlorine
(.68)(x lbs/day hypochlorite) = 5.00 lbs/day Cl_2

$$x \text{ lbs/day hypochlorite} = \frac{5.00}{.68}$$

$$x = 7.35 \text{ lbs/day hypochlorite}$$

Practice Problem 1.8

8) At a treatment plant to maintain a 0.2 mg/l chlorine residual throughout the distribution system, a chlorine dosage of 1.2 mg/l is required at the clear well. If the average daily flow reaches 0.45 MGD, how many lbs/day of hypochlorite with 68% available chlorine must be added?

Ans. _____

DATE: _____

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SECTION II - PROCESS CALCULATIONS

Chapter 1 - Chlorination Assignment Sheet

(To be returned to instructor)

1. Assume a residual of 0.2 mg/l is to be maintained after a contact time of 10 minutes in water having a 10 minutes demand of 0.4 mg/l.

   A. What dosage of liquid chlorine will be required to treat 1 million gallons?

   B. How many pounds will be required?

   C. Using the answer for B, calculate the dosage rate required to treat 6 m.g.d.

2. A chlorinator is set to feed chlorine at the rate of 160 lbs. per 24 hours into a 30 m.g.d. flow. The residual chlorine is 0.1 mg/l. What is the chlorine demand of the water?

3. HTH containing 70% available chlorine is used to treat a basin of water which is 70 feet long, 40 feet wide and 10 feet high. A) How much water is treated? B) How many pounds of chemical are needed for 0.5 mg/l dosage?

4. Determine the dosage if 6 m.g.d. of water is chlorinated with liquid chlorine at a rate of 120 lbs/day.

5. Determine the dosage if 6 m.g.d. of water is chlorinated with a solution of 30 pounds of hypochlorite.

CHEMICAL COAGULATION AND SEDIMENTATION

A. CALCULATING FEED RATE

> Chemical coagulation consists of treating the water with certain chemicals to bring non-settleable particles together into larger heavier masses of solid material, which are then comparatively easy to remove. These masses are called "Floc".

You will recall from our discussion of chlorination that we had used a formula in our discussion on feed rate. We use this same formula in calculating the feed rate of the other chemicals used in coagulation.

Chemical Used lbs/day = (dose mg/l) (flow-mgd)(8.34 lb/gal)

or if you prefer

Chemicals Used in Lbs/day = (dose mg/l)(flow mgd)(8.34 lb/gal)

Example 1:

A water treatment plant operates at a rate of 2 mgd. The dosage of alum is 35 ppm or mg/l; How many pounds of alum are used a day?

35 x 2 x 8.34

lbs/day = (dose mg/l)(flow-mgd)(8.34 lb/gal)

lbs/day = (35)x(2)x(8.34)

= (70)(8.34)

= 584 lbs/day of Alum

Example 2:

The Village of Madrid treatment plant has a flow rate of 560,000 g.p.d. The ferric chloride used in coagulation is added at a dosage of 1.8 ppm. How many pounds of ferric chloride will be used per day?

lbs/day = (dose mg/l)(flow-mgd)(8.34 lbs/day)

lbs/day = (1.8 mg/l)(0.56)(8.34 lbs/day)

= 1.0 x 8.34

= 8.34 lbs/day Ferric Chloride

Practice Problems 2.1 - 2.2

1) Given: Dose of soda ash = 0.8 mg/l
Flow = 2.6 mgd
Find: lbs/day of soda ash Ans. _____

2) Given: Dose of Sodium Aluminate = 1.4 mg/l
Flow rate = 3.6 m.g.d.

Ans. _____

Find: lbs/day of Sodium Aluminate

B. SOLUTION STRENGTHS

In calculating solution strengths, follow the procedure used below.

Example 3:

Five lbs. of Alum are added to 95 lbs. of water.

$$\frac{5}{5 + 95} \times 100 = 5\% \text{ Solution}$$

This same concept also applies to other solution strengths. For example, consider the following: Twenty pounds of alum are added to 80 pounds of water.

$$\frac{20}{20 + 80} \times 100 = 20\% \text{ Solution}$$

You will also notice that we are adding pounds of chemicals to pounds of water. You will recall that 1 gallon of water = 8.34 pounds. Then by multiplying the number of gallons by the 8.34 factor, we can find pounds.

Example 4:

30 pounds of soda ash are added to 60 gallons of water. What is the solution strength?

$$\frac{30 \times 100}{30 + (60 \times 8.34)} = \underline{5.7\% \text{ Solution}}$$

or you can do the same calculation this way:

60 x 8.34 = 500.4 lbs. of water

500.4 lbs. of water + 30 lbs. soda ash = 530.4 lbs. solution

$$\frac{30 \times 100}{530.4} = \underline{5.7\% \text{ Solution}}$$

Suppose we have hypochlorite solution.

Example 5:

To make a 5% chlorine solution from HTH containing 60% available chlorine we would need how many pounds of HTH per 100 pounds of water.

$$\frac{5}{0.60} = 8.3 \text{ lbs. HTH per 100 lbs. of water}$$

C. PROBLEMS IN CHANGING SOLUTION STRENGTH

Example 6: Given a 100 gallon tank containing 50 gallons of 5% solution, how many pounds of chemical must be added to result in an 8% strength solution with a full tank (100 gallons)?

$$\frac{8}{100} \times 100 \text{ gallons} \times 8.34 \text{ lb/day} =$$

$$\frac{8}{\cancel{100}} \times \frac{\cancel{100}^{1}}{1} \times 8.34 \text{ lb/day} = 66.7 \text{ lbs. of chemical desired in a full tank}$$

Example 6 continued:

$\frac{5}{100}$ x 50 gallons x 8.34 lbs/day =

$\frac{2085}{100}$ = 20.85 lbs. chemical in tank now

The problem asks for the amount of chemical which must be added to the tank when filled with water. Therefore, we subtract the pounds of chemical now in the tank from the pounds of chemical in a full tank. This will give us the number of pounds of chemical to be added.

66.7 lbs. of chemical desired
- 20.8 lbs. of chemical now
45.9 lbs. of chemical to be added to tank plus fill with water

Example 7:

Given a 30 in. deep tank holding 12 in. of solution, which contains 2 ounces (oz) of chemical per inch of depth. It is desired to have a full tank of solution having a strength of 3 oz. per inch. How many oz. of chemical must be added when the tank is filled.

30 in. x 3 oz/in = 90 oz. of chemical in a full tank
12 in. x 2 oz/in = 24 oz. of chemical in a tank now.

To find out how much chemical to add to the tank we subtract.

90 oz. of chemical in a full tank
24 oz. of chemical in a tank now
66 oz. of chemical to be added and fill with water

Example 8:

A 60 gallon tank contains 20 gallons of 4% strength solution. It is desired to have a full tank of 5% strength solution. A quantity of 15% stock solution if available as make-up. How many pounds of stock solution must be added to reach the desired solution strength.

$\frac{4}{100}$ x 20 gal. x 8.34 lbs/day = 6.67 lbs. of chemical in tank now

$\frac{5}{100}$ x 60 gal. x 8.34 lbs/day = 25.02 lbs. of chemical desired in a full tank.

To find how much chemical to add before filling the tank with water, we subtract.

25.02 lbs. of chemical
- 6.67 lbs. of chemical
18.35 lbs. of chemical to be added

The chemical to be added is available as a 15% stock solution. Therefore, we have 2 more steps to perform in solving our problem.

Step 1:

$\frac{18.35}{0.15}$ = 122.3 lbs. of 15% stock solution which must be added to the tank before adding water.

In the first step of this problem we found the amount of pounds of chemical to be added before we fill the tank with water. However, the problem states that a 15% stock solution is to be used. Therefore, we divide the percent stock solution into the pounds of chemical. The answer is given in pounds of 15% stock solution. There is, 1 more step.

Step 2: $\frac{122.3}{8.34}$ = 15 gallons of stock solution to be added

We divide the pounds of 15% stock solution by 8.34 lbs/gal. This is done because we are dealing in gallons of solution and have to convert pounds of solution to gallons of solution (we assumed the solution weight was equal to that of water).

Practice Problem 2.3 - 2.4

3. Calculate % of strength of a solution if 10 pounds of chemical is added to 5 gallons of water.

Ans. _____

4. Flow is 15.2 m.g.d. and the dose of Alum is 2 p.p.m. How many pounds of Alum will be used in 30 days?

Ans. _____

D. DETENTION TIME

Example 9: A sedimentation tank handles a flow of 0.9 m.g.d. The tank is 40 x 15 x 10 feet and rectangular in shape. What is the detention time?

Given: Flow - 0.9 m.g.d.
Length - 40 ft.
Width - 15 ft.
Depth - 10 ft.

Find: Detention Time, Hours

$$\text{Detention Time, Hours} = \frac{(\text{tank volume, cu. ft.})(7.48 \text{ gal/cu.ft.})(24 \text{ hr/day})}{\text{Flow, gal/day}}$$

$$\text{Tank Volume (cu. ft)} = \text{ft.} \times \text{ft.} \times \text{ft.}$$
$$= 40 \times 15 \times 10$$
$$= 6000 \text{ cu. ft.}$$

Using the above formula for detention time, we "plug in" what is given in the problem:

$$\text{Detention Time, Hours} = \frac{(\text{tank vol. cu. ft.})(7.48 \text{ gal/cu.ft.})(24 \text{ hr/day})}{\text{Flow in gal/day}}$$

$$= \frac{(6000 \text{ cu.ft.})(7.48 \text{ gal/cu.ft.})(24 \text{ hr/day})}{900{,}000 \text{ gal/day}}$$

$$= \frac{1{,}077{,}120}{900{,}000 \text{ gal/day}}$$

$$= 1.19 = \underline{1.2 \text{ hours}}$$

Example 10: A circular clarifier handles a flow of 12 m.g.d. The clarifier is 50 feet in diameter and 8 feet deep. Find the detention time.

Given: Flow = 12 m.g.d.
Diameter = 50 feet
Depth = 8 feet

Find: Detention Time, Hours

$$\text{Detention Time, Hours} = \frac{(\text{tank volume cu. ft.})(7.48 \text{ gal/cu.ft.})(24 \text{ hr/day})}{\text{Flow, gal/day}}$$

$$\text{Tank Volume, Cu. Ft.} = (\text{area, sq. ft.})(\text{depth, ft.})$$

$$\text{Clarifier Area} = 0.785 \, (\text{diameter, ft.})^2$$
$$= 0.785 \, (50 \text{ ft.})^2$$
$$= 1962.5 \text{ sq. ft.}$$
or
1963 sq. ft.

Tank Volume = (area, sq.ft.) (depth, ft.)
= (1,963 sq. ft.) (8ft.)
= 15,704 cu. ft.

$$\text{Detention Time Hours} = \frac{\text{(tank volume, cu. ft.) (7.48 gal/cu. ft.) (24 hr/day)}}{\text{flow, gal/day}}$$

$$= \frac{\text{(15,704 cu.ft.) (7.48 gal/cu. ft.) (24 hr/day)}}{\text{12,000,000 gal/day}}$$

$$= \frac{2,819,182}{12,000,000 \text{ gal/day}}$$

= 0.2349

= 0.24 Hours

Practice Problems 2.5 - 2.6

5. A sedimentation tank handles a flow of 6.5 mgd. The tank is 60 feet x 20 feet x 15 feet and rectangular in shape. What is the detention time?

Ans. _____

6. A circular clarifier handles a flow of 0.75 mgd. The clarifier has a 20 foot radius, and a depth of 10 feet. Find the detention time.

Ans. _____

DATE: _____

STUDENTS NAME: _____

SECTION II

CHAPTER II ASSIGNMENT SHEET

(To be returned to instructor)

1. If you were required to add 0.6 mg/l of flouride to your water supply, how many pounds of sodium flouride would you add per hour, if you were treating water at the rate of 1,250 gal/min?

2. Stock line is 85% calcium oxide. How many pounds of stock must be fed per day to treat a flow of 5 m.g.d., with a dose of 3 p.p.m.?

3. If a flow is 650,000 gal/day and a dose of 2 mg/l is applied, how many pounds of alum will be used in 30 days?

4. Fifty gallons of alum solution containing 2 pounds per gallon are added to 95 gallons of solution containing 3 pounds per gallon. How many pounds of alum are in each gallon of mixture?

5. A plant has a 100 gallon tank containing 50 gallons of 5% solution of synthetic polymer. Calculate how many pounds of polymer would have to be added to result in a 9% solution strength with a full tank (100 gallons)?

6. A sedimentation basis is 120 feet long, 30 feet wide, and 20 feet deep at the flow line. The tank shape is rectangular, and the flow is 5 m.g.d. What is the detention time?

7. A settling basin at a plant serving 5,000 people with a flow of 500,000 g.p.d. is 40 feet x 20 feet x 10 feet. What is the detention time in hours?

8. A circular clarifier is 20 feet in diameter with a depth of 8 ft. 3 in. The flow is 314,000 gal per day. Find the detention time.

(OVER)

9. What will be the detention time in a sedimentation basin having a diameter of 24 feet, a depth of 8 feet and a flow of 0.3 m.g.d.?

SECTION II CHAPTER III

FILTRATION

A. RATE OF FILTRATION

Example 1: A filter box is 25 feet x 30 feet, which is also the sand area. If the influent valve is shut, the water drops 3.5 inches per minute. What is the rate of filtration in mgd?

Given: Filter Box = 25 x 30 feet
Water drops = 3.5 inches per minute

Find : Rate of filtration in m.g.d.

Step 1: 25 x 30 feet = 750 sq.ft.
-Dividing 3.5 by 12 to find feet

Before we find cubic feet we have to convert 3.5 into feet.

3.5 ÷ 12 = 0.29 feet.
750 sq.ft. x 0.29 ft. = 217.5 cu.ft.

We now need to convert cubic feet to gallons.

Step 2: 217.5 cu.ft. x 7.48 gal/cu. ft. = 1,626.9 gal/min

Step 3: The problem asks for the rate of filtration in M.G.D. - to find M.G.D. simply multiply the number of gallons per minutes by the number of minutes per day.

1626.9 gal/min x 1440 min/day = 2.34 M.G.D.

Practice Problems 3.1 - 3.2

1. A filter plant has 3 filters, each measuring 10 feet long x 7 feet wide. One filter is out of service, and the other 2 together are capable of filtering 280 g.p.m. How many gallons per square foot per minute will each filter?

Ans. _____

2. A filter box is 35 feet x 25 feet which also includes the sand area. If the influent valve is shut, the water drops 4.5 in. per minute. What is the rate of filtration in m.g.d.?

Ans. _____

NOTE:

Remember to perform the calculations on a step by step basis and to break down the problem into what is given and what we are asked to find.

B. BACKWASH RATE

Example 2: A filter has the following dimensions:

Length = 35 feet
Width = 25 feet
Depth of filter media = 28 inches

Assuming a backwash rate of 16 gallons per square foot per minute is recommended and 10 minutes of backwash is required, calculate the amount of water in gallons required for each backwash.

Given: Length - 35 feet
Width - 25 feet
Depth of filter media 28 inches
Rate 16 gal/sq.ft./min.
10 min. backwash time

Find: Amount of water in gallons required

Step 1: Area of filter = 35 ft. x 25 ft. = 875 sq. ft.

Step 2: Gallons of water used per square foot of filter =
16 gal/sq.ft./x 10 = 160 gal/sq. ft.

Step 3: Gallons required = 160 gal/sq. ft. x 875 sq. ft.
= 140,000 gal. required for backwash - Answer

You will recall from Chapter IX on conversions that when we multiply minutes times 1/minutes as we did in Step 2, that the minutes cancel each other; as do the square feet in Step 3.

Example 3: A filter has the following dimensions:

Length - 10 feet
Width - 7 feet
Washwater pump is rated 2000 gpm.

What is the rate of rise in the filter?

Given: Length 10 feet
Width 7 feet
Washwater pump rate 2000 g.p.m.

Find: Rate of rise in the filter

Step 1: Area of filter = 10ft. x 7 ft. = 70 sq. ft.

Step 2: 2000 gpm ÷ 70 sq. ft. = 28.6 gal/sq.ft./min.

Step 3: 28.6 gal/sq.ft./min. ÷ 7.48 gal/cu.ft. = 3.8 cu.ft./sq.ft./min

Step 4: 3.8 cu.ft./sq. ft./min. = 3.8 ft./min.- Answer

Practice Problems 3.3 - 3.4

3. A filter having an area of 300 square feet is ready to be backwashed. Assuming a rate of backwash of 15 gallons per square foot per minute and 8 minutes of backwash is required. What is the amount of water in gallons required for each backwash?

Ans. _____

4. A filter has the following dimensions:

Length = 28 feet
Width = 21 feet
Filter media depth = 46 inches

Assuming that the wash water pump is rated at 2500 g.p.m., what is the rate of rise in the filter?

Ans. _____

STUDENTS NAME: _____ DATE: _____

SECTION II CHAPTER III
ASSIGNMENT SHEET

(To be returned to instructor)

1. A filter measures 26 feet x 15 feet, which is also the sand area. If the influent line is shut and the water drops 2.6 inches per minute. What is the rate of filtration in M.G.D.?

2. A filter measures 26 feet by 15 feet and has a filter media depth of 36 in. Assuming a backwash rate of 15 gal/sq.ft/minute and 6 minutes of backwash is required, how many gallons of water are required for each backwash?

3. A filter has the following dimensions:

   Length = 25 feet
   Width = 12 feet
   Filter media depth = 44 inches

   Assuming that the wash water pump is rated at 1500 g.p.m., what is the rate of rise in the filter?

4. A filter plant has 6 filters, each measuring 20 feet x 15 feet wide. One filter is out of service, and the other 5 are capable of filtering 500 gpm. How many gallons per square foot per minute will each filter?

5. A filter box is 30 feet x 20 feet which also includes the sand area. If the influent valve is shut and the water drops 3.7 inches per minute, what is the rate of filtration in M.G.D.?

SECTION II CHAPTER IV

DISTRIBUTION CALCULATIONS

A. VELOCITY

> The velocity of a particle of substance is the speed at which it is moving.

It is expressed by indicating the length of travel and how long it takes to cover the distance. Velocity can be expressed in almost any distance and time units.

$$\text{Velocity (time units, m.p.h., ft/sec)} = \frac{\text{Distance Traveled, Ft.}}{\text{Time, Sec.}}$$

Example 1: A car is traveling at a rate of 280 miles per 5 hours.

$$\text{Velocity, mi/hr} = \frac{280 \text{ miles}}{5 \text{ hours}}$$

$$= 56 \text{ miles/hour}$$

> Remember, it is normal to express the distance traveled per unit time.

The velocity of water in a channel, pipe, or other condiut can be expressed in the same way.

Example 2: If the particle of water travels 600 feet in five minutes, the velocity is:

$$\text{Velocity, ft/min} = \frac{\text{Distance, ft.}}{\text{Time, min.}}$$

$$= \frac{600 \text{ ft.}}{5 \text{ min.}}$$

$$= 120 \text{ ft/min}$$

If it is desired to express the velocity in feet per second, multiply by 1 min/60 seconds.

1 minute/60 seconds is like $\frac{1}{1}$ and does not change the relative value of the answer.

It only changes the form of the answer.

$$\text{Velocity, ft/sec} = (\text{velocity, ft/min})\ (1 \text{ min}/60 \text{ sec.})$$

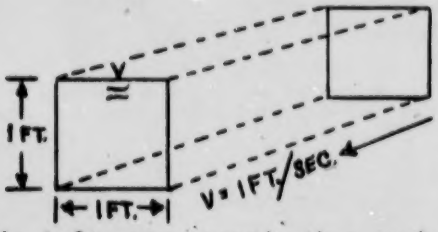
$$= \frac{120 \text{ ft} \times 1 \cancel{\text{min}}}{\cancel{\text{min}} \quad 60 \text{ sec.}}$$

$$= \frac{120 \text{ ft.}}{60 \text{ sec.}}$$

$$= 2 \text{ ft/sec.}$$

B. RATE OF FLOW

If water in a 1 foot wide conduit is one foot deep, then the cross sectional area of the conduit is 1 ft. x 1 ft. = 1 sq. ft.



If the velocity is 1 ft. per second, then each second a body of water 1 square foot in area and 1 foot long will pass a given point. The volume of this body of water will be 1 cubic foot. Since one cubic foot of water would pass by every second, the rate of flow would be equal to 1 cubic foot per second or 1 c.f.s.

To obtain the rate of flow in the above example, the velocity was multiplied by the cross sectional area. This is another important general formula.

$$Q = V \times A$$

Q = rate of flow, CFS or cu. ft./sec.
V = velocity in ft/sec
A = area in sq. ft.

Example 3: Flow in an 8 inch pipe is 500 G.P.M. What is the average velocity?

Step 1: Area, sq. ft. = 0.785 (Diameter, ft)^2

*Convert 8" to feet by dividing by 12 = $8 \div 12 = 0.66$ or 0.7 feet

$= 0.785\ (0.7)^2$
$= 0.785\ (0.49)$
$= \underline{0.38 \text{ sq. ft.}}$

We do this calculation first, because we will need the area in square feet to use in step 3 to find the velocity in ft/sec.

Step 2: $$\text{Flow, CFS} = \text{Flow, gal/min.} \times \frac{\text{cu.ft.}}{7.48 \text{ gal.}} \times \frac{1}{60 \text{ sec.}}$$

$$= \frac{500 \text{ gal.}}{\text{min.}} \times \frac{\text{cu.ft.}}{7.48 \text{ gal.}} \times \frac{1 \text{ min.}}{\text{sec.}}$$

$$= \frac{500 \text{ cu. ft.}}{448.8 \text{ sec.}}$$

$= 1.114$ CFS

The answer 1.114 is in cubis feet per second, the answer to the problem must be expressed in ft/sec.

Step 3:

$$\text{Velocity, ft/sec} = \frac{\text{Flow, cu.ft/sec.}}{\text{Area in sq. ft.}}$$

$$= \frac{1.114 \text{ ft}^3/\text{sec.}}{0.38 \text{ ft}^2}$$

$$= \underline{2.93 \text{ ft/sec.}}$$

Example 4: Flow in a 2.5 foot wide channel is 1.4 feet deep and measures 11.2 CFS. What is the average velocity?

In this problem we want to find the velocity. Therefore, we must rearrange the general formula to solve for velocity.

$$V = \frac{Q}{A}$$

Given: Q = Rate of flow, CFS = 11.2
A = Area in square feet
2.5 feed wide
1.4 feet deep

Find: V = Average Velocity

Step 1: Area in square feet = ft. x ft.
= 2.5 x 1.4
= 3.5

Step 2: Velocity, ft. sec. = $\frac{\text{flow rate, cu.ft./sec}}{\text{Area, sq. ft.}}$

$$= \frac{11.2 \text{ } \cancel{\text{cu.ft.}}/\text{sec}}{3.5 \text{ } \cancel{\text{sq.ft.}}}$$

= 3.2 ft./sec.

If you feel capable of performing the calculations in one step by plugging the numbers into the formula above, then do so or use the step by step procedure.

Practice Problems 4.1 - 4.4

1) What is the velocity in ft/min if a particle of water travels 1,500 feet in 4 minutes. What is the velocity in feet per second?

2) The flow in a 6 inch pipe is 350 G.P.M. What is the average velocity?

3) The flow in a 24 inch water main is 600 gpm. What is the average velocity?

4) What is the velocity in ft/sec of a particle of water that travels 15,000 feet in 7 minutes?

C. STORAGE TANK CALCULATIONS

Example 5: A cylindrical tank is 100 feet high, and 20 feet in diameter. How many gallons of water will it contain?

Given: Heighth - 100 feet
Diameter - 20 feet
cylindical shape

Find: Gallons of water it will contain

Step 1: We must find the volume in cubic feet.

$$\text{Volume, cu.ft.} = 0.785\ (\text{Diameter, Ft})^2\ (\text{Height, ft.})$$

$$= 0.785\ (20\ \text{ft.})^2\ (100\ \text{ft,H})$$

$$= 0.785\ (400\ \text{sq.ft.})(100\ \text{ft.,H})$$

$$= 31{,}400\ \text{cu.ft.}$$

Step 2: Our problem asks: How many gallons of water will it contain?

$$31{,}400\ \text{cu.ft.} \times 7.48\ \text{gal/cu.ft.}$$

$$= \underline{234{,}782\ \text{gallons}}$$

In example 6 we encounter a slightly different problem.

Example 6: A cyclindrical tank is to be painted inside and outside. The tank is 75 feet high and has a diameter of 30 feet. The top and bottom are flat and one gallon of paint covers 150 sq. ft. How many gallons of paint are required; if only the tank sides are to be painted not the top or bottom.

Given: cylindrical shape
height - 75 feet
diameter - 30 feet
1 gallon = 150 sq. ft.
top and bottom are flat

Find: How many gallons of paint required?

Step 1: $C = 2\pi RH$
$= 2 \times 3.14 \times 15 \times 75$

$= 7{,}065$ sq. ft.

If 1 gallon = 150 sq. ft. How many gallons will be needed?

Step 2: $7{,}065 \div 150 = 47.1$ gallons.

Step 3: Since both inside and outside are to be painted, we must double this value

$47.1 \times 2 = 94.2 \approx 95$ gallons of paint

Practice Problems 4.5 - 4.6

5) A round tank 30 feet in diameter is filled with water to a depth of 15 feet. How many gallons of water are in the tank?

6) An elevated storage tank is 80 feet high and has a diameter of 20 feet.

A. Find the square footage of the tank of both the side wall and top and bottom.
B. How many gallons of paint will be needed to paint the inside and outside? *1 gallon will cover 150 sq. ft.

D. DISINFECTION CALCULATIONS

Before being placed in service all facilities associated with the treatment and distribution of water must be disinfected. The purpose of this section is to show you how to perform the calculations involved.

Example 7: Your treatment plant has a tank and wants to disinfect is using HTH 70 percent available chlorine. The tank is 100 feet high and 20 feet in diameter. The dose to be used is 50 ppm. How many pounds do you need?

Given: Height: 100 feet
Diameter: 20 feet
50 ppm - 70% available Cl_2

Find: Pounds of HTH

Step 1: To find the cubic feet of the tank

Volume, cu.ft. = 0.785 (diameter, ft)^2 (height, ft)

= 0.785 (20 ft)^2 (height, ft)
= 0.785 (400 ft)^2 (100)
= 31,400 cubic feet

There is another formula which can be used to find the cubic feet of the tank. Feel free to use which ever you feel comfortable with.

$$X, \text{cu.ft.} = \pi R^2 \times h$$

x cu.ft. = 3.14 (10)^2 (100)
= 3.14 (100) (100)
= 3.14 (10,000)
= 31,400 cu.ft.

Step 2: Convert cubic feet to gallons (7.48 gal/cu.ft.)
31,400 cu.ft. x 7.48 cu.ft./gal = 234,872 gallons

To save ourselves some work in cancellation and converting, we are going to divide 234,872 gallons by 1,000,000 in step 3. You will recall that 1 ppm (mg/l) = 1,000,000 gallons of water, this will save us from changing ppm to pounds and back.

Step 3: 234,872 gallons ÷ 1,000,000 ppm/million gallons = 0.234 ppm

Step 4: 0.234 ppm x 8.34 lbs/mill.gal. = 1.95156 ppm/lbs/million gallons (since our answer wants to be in pounds)

You will recall from our discussion on chlorine that "the fundamental concept to keep in mind when doing hypochlorite calculations is that once we have determined how many pounds of chlorine will be required for disinfection, we will always need more pounds of hypochlorite."

Step 5: 1.95156 ppm/lbs/million x 50 ppm = 97.6 lbs. of available Cl_2.

Step 6: 97.6 lbs ÷ .70 = 139.4 lbs. HTH req.

Example 8: When it is necessary to treat 3,000 feet of 8" water line by applying enough chlorine for 70 ppm dosage, how many pounds of hypochlorite of 70% available chlorine are required?

Given: Length - 3,000 feet 70% available Cl_2
Diameter 8" 70 ppm

Find: Pounds of hypochlorite required

Step 1: Find cubic feet of pipe

Formula: x cu. ft. = $\pi R^2 \times h$
= 3.14 $(0.35)^2$ x 3000 ft.
= 3.14 (0.1225) x 3000 ft.
= 1,154 cu. ft.

*We must change 8" to feet by dividing by 12. 8 ÷ 12 = 0.66 ft. or rounding off to 0.7 feet which is the diameter. To get the radius= ½ of 0.7 or 0.35.

Step 2: Find the gallons of water in the 8" pipe.

1,154 ~~cu.ft.~~ x 7.48 gal/~~cu.ft.~~ = 8,632 gal.

Step 3: $\frac{8632 \text{ gal.}}{1{,}000{,}000 \text{ gal/ppm}} = 0.008632 = 0.0086$

Drop the two and the three because they are less than 5 according to decimal rules.

Step 4: 0.0086 ppm x 8.34 = 0.0717~~24~~ = 0.072

Rounding off decimals in the answer.

Step 5: 0.0072 ~~ppm~~/lb x 70 ~~ppm~~ = 5.04 lbs of available Cl_2

Step 6: 5.04 ÷ .70 = 7.2 lbs of hypochlorite

Practice Problems 4.7 - 4.8

7) A wash tank holds 125,000 gallons. How many pounds of hypochlorite of 70% available chlorine is required to sterilize this tank at 50 ppm?

8) It is necessary to disinfect 4,500 feet of 6 inch water line by enough chlorine for 60 ppm dosage. How many pounds of HTH at 70% strength is required?

DATE: _____

STUDENTS NAME: _____

SECTION II CHAPTER IV
ASSIGNMENT SHEET

(To be returned to instructor)

1. Water is pumped from a well at the rate of 450 gpm through a 6 inch main. What is the velocity of flow (in feet per second) in the line?

2. A basin is 120 ft x 30 ft x 20 ft. The flow is 3 mgd. Calculate the A) Volume, B) Detention time, C) The flow through velocity.

3. A sedimentation basin is 120 ft long x 30 ft wide x 20 ft deep at the flow line. The shape is rectangluar. The flow is 5 mgd. Calculate A) Volume, B) Detention time, C) Flow through velocity.

4. A new tank is 80 x 30 x 20 feet. How many pounds of HTH 70% available chlorine is required to disnifect the tank at 50 mg/l?

5. Your water district wants to disinfect a tank using HTH 70% available chlorine. The tank is 75 feet high and 30 feet in diameter. The dose to be used is 60 ppm. How many pounds of HTH do you need?

SECTION II CHAPTER V

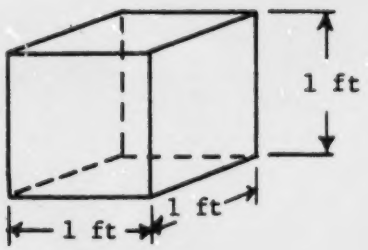
FORCE, PRESSURE, AND HEAD

In order to study the forces and pressures involved in fluid flow, it is first necessary to define the terms used.

Force:	The push exerted by water on any surface being used to confine it. Force is usually expressed in pounds, tons, grams, or kilograms.
Pressure:	The force per unit area. Pressure can be expressed in many ways, but the most common term is pounds per square inch (psi).
Head:	Vertical distance from the water surface to a reference point below the surface. Usually expressed in feet or meters.

An example should serve to illustrate these terms.

If water were poured into a one-foot cubical container, the force acting on the bottom of the container would be 62.4 pounds.



The pressure acting on the bottom would be 62.4 pounds per square foot. The area of the bottom is also 12 in x 12 in = 144 in^2. Therefore, the pressure may also be expressed as:

$$\text{Pressure, psi} = \frac{62.4 \text{ lb}}{\text{sq ft}} = \frac{62.4 \text{ lb/sq ft}}{144 \text{ sq in/sq ft}}$$

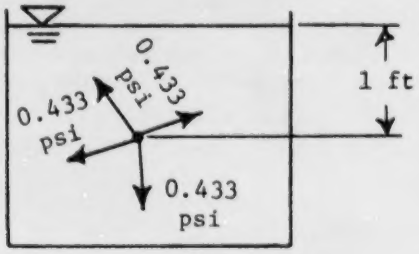
$$= 0.433 \text{ lb/sq in}$$

$$= 0.433 \text{ psi}$$

Since the height of the container is one foot, the head would be one foot.

*Taken from Mathematics for Water and Wastewater Treatment Plant Operators, South Dakota Department of Environmental Protection

The pressure in any vessel at one foot of depth or one foot of head is 0.443 psi acting in any direction.



If the depth of water in the previous example were increased to two feet, the pressure would be:

$$p = \frac{2\ (62.4\ \text{lb})}{144\ \text{sq in}} = \frac{124.8\ \text{lb}}{144\ \text{sq in}} = 0.866\ \text{psi}$$

Therefore we can see that for every foot of head the pressure increases by 0.433 psi. Thus, the general formula for pressure becomes:

$$p, \text{psi} = 0.433\ (H, \text{ft})$$

H = feet of head

p = pounds per square inch of pressure

$$P, \text{lb/sq ft} = 62.4\ (H, \text{ft})$$

H = feet of head

P = pounds per square foot of pressure

We can now draw a diagram of the pressure acting on the side of a tank. Assume a four-foot deep tank. The pressures shown on the tank are gage pressures. These pressures do not include the atmospheric pressure acting on the surface of the water.

If the pressure in psi were used, the problem would be similar:

Force, lb = (Pressure, lb/sq in) (Area, sq in)
= 0.866 psi x 48 in x 60 in
= 2494 lb*

The general formula, then, for finding the total force acting on a side wall of a tank is:

$$F = 31.2 \times H^2 \times L$$

F = force in pounds

H = head in feet

L = length of wall in feet

31.2 = constant with units of lbs/cu ft and considers the fact that the force is exerted at H/2 or half the depth of the water.

Example 1 Find the force acting on a five-foot long wall in a four-foot deep tank.

Force, lb = 31.2 (Head, ft)^2 (Length, ft)

= 31.2 lb/cu ft x (4 ft)^2 x 5 ft

= 2496 lbs

Occasionally an operator is warned: Never empty a tank during periods of high groundwater. Why? The pressure on the bottom of the tank caused by the water surrounding the tank will tend to float the tank like a cork if the upward force of the water is greater than the weight of the tank.

$$F = 62.4 \times H \times A$$

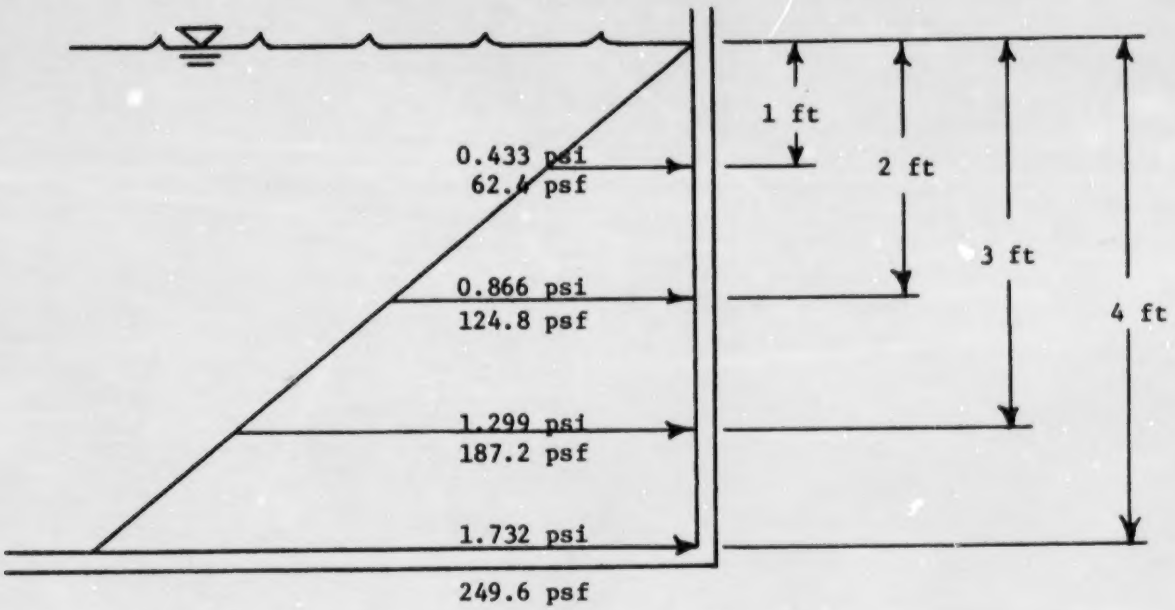
F = upward force in pounds

H = head of water on tank bottom, in feet

A = area of bottom of tank in square feet

62.4 = a constant with units of lbs/cu ft

* Difference in answer due to rounding off of decimal points.



$p_0 = 0.433 \times 0 = 0.0$ psi $\qquad P_0 = 62.4 \times 0 = 0.0$ lb/sq ft

$p_1 = 0.433 \times 1 = 0.433$ psi $\qquad P_1 = 62.4 \times 1 = 62.4$ lb/sq ft

$p_2 = 0.433 \times 2 = 0.866$ psi $\qquad P_2 = 62.4 \times 2 = 124.8$ lb/sq ft

$p_3 = 0.433 \times 3 = 1.299$ psi $\qquad P_3 = 62.4 \times 3 = 187.2$ lb/sq ft

$p_4 = 0.433 \times 4 = 1.732$ psi $\qquad P_4 = 62.4 \times 4 = 249.6$ lb/sq ft

The average pressure acting on the tank wall is 1.732 psi/2 = 0.866 psi, or 249.6 psf/2 = 124.8 psf.

If the wall were five feet long, the pressure would be acting over the entire 20 square foot (5 ft x 4 ft) area of the wall. The total force acting to push the wall would be:

Force, lb = (Pressure, lb/sq ft)(Area, sq ft)

= 124.8 lb/sq ft x 20 sq ft

= 2496 lbs

Example 2: Find the upward force on the botton of an empty tank caused by a ground water depth of 8 feet above the tank bottom. The tank is 20 ft. wide and 40 ft. long.

Force, lb. = 62.4 (Head, ft)(area, sq ft)

= 62.4 lb/cu ft x 8 ft x 20 ft x 40 ft

= 399,400 lb

We can then summarize our discussion of Force, Pressure and Head into 3 basic types of computation.

Since 1 cubic foot of water weighs 62.4 pounds, a column of water 1 foot high and 1 square inch in cross section will weigh 0.433 pounds, that is

$$\frac{(1\ \text{ft.})(1\ \text{in.})^2\ (62.4\ \text{lb/ft})^3}{144\ \text{in}^2/\text{ft}^2}$$

Therefore:

A) Pressure (in P.S.I.) = 0.422 x Head (in ft.)
The reciprocal of 0.433 is $\frac{1}{0.433}$, or 2.31, Therefore:

B) Head (in ft.) = 2.31 x pressure (in P.S.I.)

The total force exerted on an area is equal to pressure times area, or area times depth to which the center of area is submerged times the weight of a cubic unit of water. Therefore:

C) Total Force (in lbs.) = pressure (in psi) x area (in sq.ft.)

= area (in sq. ft.) x head (in ft.)

at the center of area x 2.4

Practice Problems 5.1 - 5.2

1) Convert 50 feet of Head to pounds per square inch.

ANS. _____

2) Calculate the Force (in lbs.) acting on a four-foot long wall in a ten-foot deep tank.

ANS _____

HORSEPOWER

Horsepower is a unit of power. One horsepower is defined as 33,000 ft lbs/min or 746 watts.

Work is also done by lifting water. If the flow from a pump is converted to a weight of water and multiplied by the vertical distance it is lifted, the amount of work or power can be obtained.

This is called water horsepower.

$$\text{Water, HP} = \frac{(\text{Flow g.p.m. (H, FT.)}}{3960*}$$

Since pumps are not 100% efficient (they cannot transfer all the power put into them), the HP supplied to a pump is greater than the water horsepower. HP supplied to the pump is called brake horsepower.

$$\text{Brake, HP} = \frac{\text{Flow, g.p.m. x Ft.}}{3960 \times Ep}$$

Ep = efficiency of pump (usual range 50-85%, Depending on type and size of pump)

Motors are not 100% efficient, therefore the power supplied to the motor is greater than the motor transmits.

$$\text{Motor, HP} = \frac{\text{Flow, g.p.m. H, Ft.}}{3960 \times Ep \times Em}$$

Em = efficiency of motor (usual range 80-95% depending on type and size of motor)

The above formulas have been developed for the pumping of water, having a specific gravity of 1.0. If other liquids are to be pumped, the formulas must be multiplied by the specific gravity of the liquid.

Practice Problems 5.3 - 5.4

3) Calculate the water HP required to pump 5.[illegible] MGD of water against a total head of 23 feet with a 90% efficient pump and 88% efficient motor.

ANS. _____

4) A flow of 500 gpm of water is to be pumped against a total head of 100 feet by a pump with an efficiency of 70%. What is the pump HP?

ANS. _____

*8.34 lb/gal x $\frac{\text{HP}}{33{,}000 \text{ ft/lbs/min}} = \frac{1}{3960}$

STUDENTS NAME: _____ DATE: _____

SECTION II CHAPTER V

ASSIGNMENT SHEET

(To be returned to instructor)

1. What is the pressure in psi exerted on the bottom of a tank if the head of water is 30 feet?

2. A cylindrical tank with a diameter of 40 feet and a height of 85 feet is to be painted inside and outside. Both the top and the bottom are flat.

   A) How many gallons of water does the tank hold when in use?
   B) How many gallons of paint will be needed? (1 gallon = 150 sq.ft.)

3. A pump with a capacity of 95 g.p.m. will pump how many gallons of water in 2 hours and 20 minutes?

4. A tank is 80 x 30 x 20 feet and rectangular in shape. A pump fills it in 8 hours. How many g.p.m. does the pump deliver?

5. Calculate the horsepower required to pump 3.1 MGD of water per day against a Head of 55 feet with a 92% efficient pump and a 89% efficient meter.

SECTION III-APPENDIX

A. BASIC UNITS

1. ENGLISH SYSTEM

Linear	1 foot (ft)	=	12 inches (in)
	1 yard (yd)	=	36 inches (in)
	1 yard (dy)	=	3 feet (ft)
	1 mile (mi)	=	5280 feet (ft)
	1 mile (mi)	=	1760 yards (yds)
Area	1 square foot (ft^2)	=	144 square inches (in^2)
	1 square yard (yd^2)	=	9 square feet (ft^2)
	1 acre (A)	=	43,560 square feet (ft^2)
Volume	1 cubic foot (ft^3)	=	1728 cubic inches (in^3)
	1 cubic yard (yd^3)	=	27 cubic feet (ft^3)
	1 cubic foot (ft^3)	=	7.48 gallons (gal)
	1 gallon (gal)	=	4 quarts (qts)
Weight	1 pound (lb)	=	16 ounces (oz)
Water	1 cubic foot water	=	62.4 pounds (lbs)
	1 gallon water	=	8.34 pounds (lbs)

1 part per million (ppm) = 1 gallon per million gallons (1 gal/MG)

1 part per million water = 8.34 lbs. per million gallons (lbs/MG)

II. METRIC SYSTEM

Linear	1 centimeter (cm)	=	10 millimeters (mm)
	1 meter (m)	=	100 centimeters (cm)
	1 kilometer (km)	=	1000 meters (m)
Area	1 square meter (m^2)	=	10,000 square centimeters (cm^2)
Volume	1 milliliter (ml)	=	1 cubic centimeter (cc)
	1 milliliter (ml)		0.001 liter (l)
	or		
	1 cubic centimeter (cc)		0.001 liter (l)
	1 liter (l)	=	1000 milliliters (ml)
			or
			1000 cubic centimeters (cc)
	1 kiloliter (kl)	=	1000 liters (l)
Weight	1 milligram (mg)	=	0.001 gram (gm)
	1 gram (gm)	=	1000 milligrams (mg)
	1 kilogram (kg)	=	1000 grams (gm)

1 part per million (ppm) = 1 milligram per kilogram (mg/kg)

1 part per million water = 1 milligram per liter (mg/l)
1 part per million water = 1 gram per cubic meter (g/m^3)

III. WEIGHTS, MEASURES AND PHYSICAL UNITS

The English system of weights and measures is used in the treatment plant, but in the laboratory the metric system is universal. The basic units of both systems and their relationships follow.

Basic Units.

Linear
1 inch (in) = 2.54 centimeters (cm)
1 foot (ft) = 12 in
1 meter (m) = 39.37 in = 3.28 ft = 100 cm
1 centimeter = 10 millimeters (mm)

Area
1 square inch (sq in) (in^2)
1 square foot (sq ft) (ft^2) = 144 in^2
1 acre (a) = 43560 ft^2
1 square mile = 640 acres

Volume
1 cubic inch (cu in) (in^3)
1 cubic foot = 1728 in^3 = 7.48 gal
1 gallon = 4 quarts (qt)
1 gallon = 3.785 liters (l)
1 liter = 1000 milliliters (ml)
1 quart = 946 ml

Weight
1 pound (lb) = 16 ounces (oz)
1 pound = 453.6 grams (gm)
1 pound = 7000 grains (gr)
1 kilogram (kg) = 1000 grams = 2.205 pounds

Physical units must be kept in mind when calculations with concrete numbers are performed in order to give meaning to the answer.

Some frequently used rates and units are:

1 cubic foot of water weighs 62.4 pounds
1 gallon of water weighs 8.34 pounds
1 liter of water weighs 1,000 grams
1 ounce weighs 28.35 grams
1 milliliter of water weighs 1 gram
1 ppm = 1 lb per million lbs
1 ppm = 8.34 lb per million gallons of water
1 ppm = 1 milligram per liter (closely) for water
1% = 10,000 ppm
1 oz per ft^3 = 1 gram per liter
1 million gal per day = 1.55 cu ft per sec
1 million gal per day = 694 gal/min
1 cu ft per sec = 450 gal/min

Temperature Water freezed at 32° and boils at 212° Fahrenheit
°F = 9/5 °C + 32
°C = (°F - 32) 5/9

B. CONVERSION FACTORS

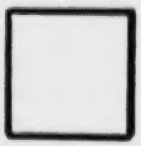
Multiply	By	To Obtain
Centimeters	0.3937	Inches
Centimeters	0.01	Meters
Centimeters	10	Millimeters
Cubic Feet	1728	Cubic in.
Cubic Feet	0.02832	Cu. Meters
Cubic Feet	7.48	Gallons
Cubic Feet	28.32	Liters
Cu.Ft/min	0.1247	Gal/Sec.
Cu.Ft/min	0.4720	Liters/sec.
Cu.Ft/min	62.43	Lbs.of water/min
Cu.Ft/sec.	0.646	M G D
Cu.Ft/sec.	448.8	Gal/Min.
Decigrams	0.1	Grams
Deciliters	0.1	Liters
Decimeters	0.1	Meters
Feet	30.48	Centimeters
Feet of Water	62.4	Lbs/Sq.Ft.
Feet of Water	0.4335	Lbs/Sq. In.
Feet/Min.	0.01667	Feet/Sec.
Gallons	0.1337	Cu. Ft.
Gallons	231	Cu. In.
Gallons	3.785	Liters
Gal. Water	8.34	Lbs. Water
Gr./U.S. Gal.	17.1	PPM/mg/l
Gr/U.S. Gal.	142.86	Lbs/Mil. Gal.
H.P.	0.7457	Kilowatts
H.P.	745.7	Watts
Inches	2.540	Centimeters
Inches of Water	5.202	Lbs/Sq. Ft.
Inches of Water	0.03613	Lbs/Sq. In.

Multiply	By	To Obtain
Meters	100	Centimeters
Meters	3.281	Feet
Meters 1	39.37	Inches
Meters	1.094	Yards
Meters/min	1.667	Cm/sec.
Meters/min	3.281	Feet/min.
Meters/min	0.05468	Feet/sec.
MGD	1.54723	Cu.Ft./sec.
Ounces	16	Grams
Ounces	137.5	Grains
Ounces	0.0625	Pounds
Ounces	28.349527	Grams
Ounces/Fluid	1.805	Cu. In.
PPM	0.0584	Grains/U.S. Gal.
PPM	8.34	Lbs/Million Gal.
Pounds	16	Ounces
Pounds	256	Grams
Pounds	7000	Grains
Pounds	453.59	Grams
Lbs/Water	0.01602	Cu. Ft.
Lbs/Water	27.68	Cu. In.
Lbs/Water	0.1198	Gallons
Lbs/Sq. In.	2.307	Feet of Water
1/ sq.ft/gal/min	8.0208	Overflow Rate ft/hr.
Tons of water/24 hrs.	83.333	Lbs/Water/hr
Tons of water/24 hrs.	0.16643	Gals./Min.
Tons of water/24 hrs.	1.3349	Cu.Ft./Hr.
* Gal.Water/Min	6.0086	Tons Water/ 24 Hours

C. DIMENSIONAL. FORMULAS

1. Perimeters

Square:



$P = s + s + s + s$

Rectangle

$P = L + W + L + W = 2L + 2W$

Circle:

$C = 2\pi R$ $C = \pi D$

II. Areas

Triangle:

$A = \frac{1}{2} B \times H$

Square:

$A = S^2$

Rectangle:

$A = L \times W$

Circle:

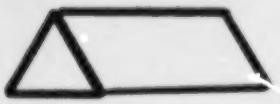
$A = \pi R^2$ $A = \pi \frac{D^2}{4}$

III. Volumes

Cube:

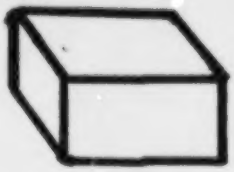
$V = S \times S \times S = S^3$

Triangular:



$$V = \frac{1}{2} B \times H \times L = A \times L$$

Rectangular Tank:



$$V = L \times W \times H$$

Cylinder:

$$V = \pi R^2 h = \pi \frac{D^2}{4} h$$

D. PROCESS FORMULAS

I. Chlorination

A. Dosage (mg/l) = Demand (mg/l) + Residual (mg/l)
B. Demand (mg/l) = Dosage (mg/l) - Residual (mg/l)
C. Residual (mg/l) = Dosage (mg/l) - Demand (mg/l)
D. Mg/l (PPM) = $\frac{\text{Lbs. of chemical used}}{\text{water flow in MGD} \times 8.34}$
E. Lbs. of Chemical Required Per Day = 8.34 x Flow (MGD) x Dose in ppm (mg/l)

II. Chemical Coagulation

A. Grains per Gallon = $\frac{\text{Lbs. of Chemical Used}}{\text{Water Flow (MGD)} \times 143}$

B. Mg/l (PPM) = $\frac{\text{Lbs. of Chemical Used}}{\text{Water Flow (MGD)} \times 8.34}$

C. Lbs. of Chemical Required Per Day = 143 x Water Flow (MGD) x Dose in Grains/Gal.

D. Lbs. of Chemical Required Per Day = 8.34 x Water Flow (MGD) x Dose in mg/l (PPM)

III. Sedimentation

Detention Time (Hours) = $\frac{\text{(Tank Volume, Cu.Ft.) (7.48 gal/cu.ft) (24 hr/day)}}{\text{Flow, gal/day}}$

IV. Velocity and Rate of Flow

A. Velocity Ft/Min = $\frac{\text{Distance, Ft.}}{\text{Time}}$

B. $Q = V \times A$

Q = Rate of Flow, per CFS or cu.ft./sec.
A = Area in square feet
V = Average velocity of flow (ft/sec)

C. $V = \frac{Q}{A}$ (To find average velocity in Ft/Sec)

V. Force, Pressure, Head

A. 2.31 feet of water = 1 pound per square inch
.434 P.S.I. = 1 foot of water

B. To change feet of water to P.S.I. = Ft. of Water x .434

C. To change P.S.I. to feet of water = PSI x 2.31

D. Water, HP = $\frac{\text{(Flow gpm) (H, Ft.)}}{3960*}$

E. Brake, HP = $\frac{\text{Flow, g.p.m. (H, Ft.)}}{3960 \times E_p}$

F. Motor, HP = $\frac{\text{Flow, g.p.m. x H, Ft.}}{3960 \times E_p \times E_m}$

*8.34 lb/gal. x $\frac{\text{HP}}{33{,}000 \text{ ft/lbs/min}} = \frac{1}{3960}$

E. SOLUTIONS TO PRACTICE PROBLEMS

Section 1

Chapter 1

1.1) $10 + 15 = 15 + 10$

1.2)
$$\begin{array}{r} 1 \\ +9 \\ \hline 10 \end{array}$$

1.3)
$$\begin{array}{r} 7 \\ +8 \\ \hline 15 \end{array}$$

1.4)
$$\begin{array}{r} 1 \\ +8 \\ \hline 9 \end{array}$$

1.5)
$$\begin{array}{r} 8 \\ +5 \\ \hline 13 \end{array}$$

1.6)
$$\begin{array}{r} 2 \\ +6 \\ \hline 8 \end{array}$$

1.7)
$$\begin{array}{r} 3 \\ +5 \\ \hline 8 \end{array}$$

1.8)
$$\begin{array}{r} 1 \\ 7 \\ 5 \\ +3 \\ \hline 16 \end{array}$$

1.9)
$$\begin{array}{r} 3 \\ 9 \\ 1 \\ +7 \\ \hline 20 \end{array}$$

1.10)
$$\begin{array}{r} 1 \\ 6 \\ 2 \\ +9 \\ \hline 18 \end{array}$$

1.11)
$$\begin{array}{r} 6 \\ 4 \\ 9 \\ 1 \\ +3 \\ \hline 23 \end{array}$$

1.12)
$$\begin{array}{r} 5 \\ 4 \\ 5 \\ 6 \\ +3 \\ \hline 23 \end{array}$$

1.13)
$$\begin{array}{r} 9 \\ 1 \\ 3 \\ 7 \\ +4 \\ \hline 24 \end{array}$$

1.14)
$$\begin{array}{r} 8 \\ 5 \\ 3 \\ 7 \\ +6 \\ \hline 29 \end{array}$$

1.15)
$$\begin{array}{r} 762 \\ 349 \\ 521 \end{array}$$

15 hundreds, 12 tens, 12 ones
which is 12 hundreds, 13 tens, 2 ones
which is 16 hundreds, 3 tens, 2 ones

$= 1632$

1.16)
$$\begin{array}{r} 32 \\ 564 \\ 324 \\ 71 \end{array}$$

8 hundreds, 18 tens, 11 ones
which is 8 hundreds, 19 tens, 1 one
which is 9 hundreds, 9 tens, 1 one

$= 991$

1.17)
$$\begin{array}{r} 23 \\ 156 \\ 321 \\ 72 \end{array}$$

4 hundreds, 16 tens, 12 ones
which is 4 hundreds, 17 tens, 2 ones
which is 5 hundreds, 7 tens, 2 ones

$= 572$

1.18)
$$\begin{array}{r} 63 \\ 152 \\ 95 \\ 946 \end{array}$$

10 hundreds, 24 tens, 16 ones
which is 10 hundreds, 25 tens, 6 ones
which is 12 hundreds, 5 tens, 6 ones

$= 1256$

1.19) 93
507
212
713

which is 14 hundreds, 11 tens, 15 ones
which is 14 hundreds, 12 tens, 5 ones
which is 15 hundreds, 2 tens, 5 ones

= 1525

1.20) 8-(4-2) = 8-2 = 6

1.21) 15-6 = 9 which is the difference

1.22) 9 − 2 = 7

1.23) 7 − 4 = 3

1.24) 14 − 6 = 8

1.25) 12 − 7 = 5

1.26) 16 − 9 = 7

1.27) 11 − 5 = 6

1.28) 9 − 3 = 6

1.29) 12 − 7 = 5

1.30) 21 − 19 = 2

1.31) 324 − 49 = 275

1.32) 2376 − 1197 = 1179

1.33) 5261 − 3159 = 2102

1.34) 6 ft. 3 in. - 11 in is 5 ft. 15 in. − 11 in. = 5 ft. 4 in.

1.35) 5 ft. 6 in. - 2 ft. 9 in. is 4 ft. 18 in. − 2 ft. 9 in. = 2 ft. 9 in.

1.36) 18 ft. 2 in. - 8 ft. 8 in. is 17 ft. 14 in. − 8 ft. 8 in. = 9 ft. 6 in.

1.37) 4 x 3 = 12 - the factors are 4 and 3. The product is 12.

1.38) 4 x 3 = 12

1.39) 5 x 3 = 15

1.40) 4 x 6 = 24

1.41) 27 x 10 = 270

1.42) 9 x 10 = 90

1.43) 14 x 100 = 1400

1.44) 4 x 100 = 400

1.45) 100 x 9 = 900

1.46) 10 x 4 = 40

1.47)
$$\begin{array}{r} 15 \text{ R5} \\ 8\overline{)125} \\ 8 \\ \hline 45 \\ 40 \\ \hline 5 \end{array}$$

1.48)
$$\begin{array}{r} 1218 \text{ R4} \\ 7\overline{)8530} \\ 7 \\ \hline 15 \\ 14 \\ \hline 13 \\ 7 \\ \hline 60 \\ 56 \\ \hline 4 \end{array}$$

1.49)
$$\begin{array}{r} 1001 \text{ R2} \\ 3\overline{)3005} \\ 3 \\ \hline 00 \\ 0 \\ \hline 00 \\ 0 \\ \hline 05 \\ 3 \\ \hline 2 \end{array}$$

1.50)
$$\begin{array}{r} 855 \\ 9\overline{)7695} \\ 72 \\ \hline 49 \\ 45 \\ \hline 45 \\ 45 \\ \hline 0 \end{array}$$

1.51)
$$\begin{array}{r} 24 \\ 19\overline{)456} \\ 38 \\ \hline 76 \\ 76 \\ \hline \end{array}$$

1.52)
$$\begin{array}{r} 248 \text{ R20} \\ 32\overline{)7956} \\ 64 \\ \hline 155 \\ 128 \\ \hline 276 \\ 256 \\ \hline 20 \end{array}$$

1.53)
$$\begin{array}{r} 24 \text{ R25} \\ 27\overline{)673} \\ 54 \\ \hline 133 \\ 108 \\ \hline 25 \end{array}$$

1.54)
$$\begin{array}{r} 256 \text{ R8} \\ 2\overline{)5384} \\ 42 \\ \hline 118 \\ 105 \\ \hline 134 \\ 126 \\ \hline 8 \end{array}$$

Chapter 11

2.1) $\underline{\underline{1/3}}$

2.2) $\underline{\underline{1/4}}$

2.3) $\underline{\underline{3/16}}$

2.4) $6/6 = \underline{\underline{1}}$

2.5) $12/6 = \underline{\underline{2}}$

2.6) $26/5 = \underline{\underline{5\ 1/5}}$

2.7) $20/40 = \underline{\underline{1/2}}$

2.8) $6/36 = \underline{\underline{1/6}}$

2.9) $\frac{1}{8} + \frac{7}{8} + \frac{3}{8} = \underline{\underline{\frac{11}{8}}}$

2.10) $\frac{3}{16} + \frac{2}{16} + \frac{7}{16} = \frac{12}{16}$ which can be reduced to $\frac{3}{4}$

2.11) $\frac{7}{12} + \frac{3}{12} + \frac{1}{12} = \underline{\underline{\frac{11}{12}}}$

2.12) $\frac{1}{3}$ & $\frac{3}{4}$

Will 3 go evenly into 4 - No
Will 3 go evenly into 8 - No
Will 3 go evenly into $\underline{12}$ - Yes

2.13) $\frac{5}{6}$ & $\frac{3}{4}$

Will 4 go evenly into 6 - No
Will 4 go evenly into $\underline{12}$ - Yes

2.14) $\frac{5}{6} = \frac{5}{6}$

$$\begin{array}{r} \frac{1}{+3} = \frac{2}{6} \\ \hline \underline{\frac{7}{6}} = 1\frac{1}{6} \end{array}$$

2.15) $\frac{3}{8} = \frac{9}{24}$

$$\begin{array}{r} \frac{1}{+3} = \frac{8}{24} \\ \hline \underline{\underline{\frac{17}{24}}} \end{array}$$

2.16) $\frac{1}{9} = \frac{4}{36}$

$$\begin{array}{r} \frac{1}{+12} = \frac{3}{36} \\ \hline \underline{\underline{\frac{7}{36}}} \end{array}$$

2.17) $\frac{4}{5} = \frac{12}{15}$

$$\begin{array}{r} \frac{2}{+3} = \frac{10}{15} \\ \hline \frac{22}{15} = \underline{\underline{1\frac{7}{15}}} \end{array}$$

2.18) $\frac{1}{3} = \frac{8}{24}$

$\frac{1}{8} = \frac{3}{24}$

$$\begin{array}{r} \frac{3}{+8} = \frac{9}{24} \\ \hline \frac{20}{24} = \frac{5}{6} \end{array}$$

2.19) $\frac{1}{6} = \frac{5}{30}$

$\frac{3}{5} = \frac{18}{30}$

$$\begin{array}{r} \frac{1}{+10} = \frac{3}{30} \\ \hline \frac{26}{30} = \underline{\underline{\frac{13}{15}}} \end{array}$$

2.20) $79\frac{7}{12} = 79\frac{21}{36}$

$$\begin{array}{r} +13\frac{7}{9} = 13\frac{28}{36} \\ \hline 92\frac{49}{36} \text{ or } 93\frac{13}{36} \end{array}$$

2.21) $45\frac{3}{8} = 45\frac{6}{16}$

$+17\frac{15}{16} = 17\frac{15}{16}$

$= 62\frac{21}{16} = \underline{\underline{63\frac{5}{16}}}$

2.22) $\frac{13}{16} - \frac{3}{16} = \frac{10}{16}$ which can be reduced to $\underline{\underline{\frac{5}{8}}}$

2.23) $\frac{10}{13} - \frac{7}{13} = \underline{\underline{\frac{3}{13}}}$

2.24) $\frac{41}{50} - \frac{3}{50} = \frac{38}{50}$ which can be reduced to $\frac{19}{25}$

2.25) $\frac{3}{4} = \frac{12}{16}$

$-\frac{1}{16} = \frac{1}{16}$

$\underline{\underline{\frac{11}{16}}}$

2.26) $\frac{7}{8} = \frac{21}{24}$

$-\frac{1}{6} = \frac{4}{24}$

$\underline{\underline{\frac{17}{24}}}$

2.27) $5\frac{3}{5} = 5\frac{12}{20}$

$-2\frac{1}{4} = 2\frac{5}{20}$

$= \underline{\underline{3\frac{7}{20}}}$

2.28) $53\frac{7}{12} = 53\frac{14}{24}$

$-12\frac{3}{8} = 12\frac{9}{24}$

$= \underline{\underline{41\frac{5}{24}}}$

2.29) $9\frac{2}{7} = 8\frac{9}{7}$

$-\frac{3}{7} = \frac{3}{7}$

$= \underline{\underline{8\frac{6}{7}}}$

2.30) $9\frac{3}{8} = 9\frac{6}{16} = 8\frac{22}{16}$

$-4\frac{15}{16} = 4\frac{15}{16} = 4\frac{15}{16}$

$\underline{\underline{4\frac{7}{16}}}$

2.31) $25\frac{3}{5} = 25\frac{9}{15} = 24\frac{24}{15}$

$-14\frac{2}{3} = 14\frac{10}{15} = 14\frac{10}{15}$

$10\frac{14}{15}$

2.32) $3 \times \frac{3}{11} = \frac{9}{11}$

2.33) $5 \times \frac{3}{7} = \frac{15}{7}$ or $2\frac{1}{7}$

2.34) $5 \times \frac{1}{10} = \frac{5}{10}$ which can be reduced to $\frac{1}{2}$

2.35) $\frac{6}{5} \times \frac{3}{5} = \frac{18}{25}$

2.36) $\frac{1}{3} \times \frac{6}{9} = \frac{6}{27}$ or $\frac{2}{9}$

2.37) $\frac{8}{9} \times \frac{5}{7}$

$= \frac{8}{9} \times \frac{5}{7} = \frac{40}{63}$

2.38) $3\frac{3}{4} \times 1\frac{3}{5} = \frac{15}{4} \times \frac{8}{5} = \frac{120}{20} = 6$

2.39) $\frac{2}{3} \times 3\frac{1}{5} = \frac{2}{3} \times \frac{16}{5} = \frac{32}{15} = 2\frac{2}{15}$

2.40) $1\frac{2}{9} \times \frac{3}{22}$

$= \frac{11}{9} \times \frac{3}{22} = \frac{33}{198} = \frac{1}{6}$

Chapter III

3.1) $13.6 + 9.7 + 3.5 + 10.4 = 37.2$

3.2) $9.5 + .016 + 32.3 + 7.0312 = 48.8472$ or 48.85

3.3) $0.3 + 4.0 + 0.7 + 5.0 + 0.6 + 8.0 = 18.6$

3.4) $4.9 - 3.0 = 1.9$

3.5) $39.1 - 5.341 = \underline{33.759 \text{ or } 33.8}$

3.6) $17.369 - 4.29 = 13.079 = \underline{13.1}$

3.7) $43 \times 10 = \underline{430}$

3.8) $.35 \times 10 = \underline{3.5}$

3.9) $356.1 \times 100 = \underline{35610}$

3.10) $.809 \times 9.15 = \underline{7.40235 \text{ or } 7.40}$

3.11) $7.3 \times .9 = \underline{6.57 \text{ or } 6.6}$

3.12) $7.71 \times 3.9 = \underline{30.069 \text{ or } 30.1}$

3.13) $64.3 \div 100 = \underline{0.643 \text{ or } 0.6}$

3.14) $.146 \div 10 = \underline{0.0146 \text{ or } 0.015}$

3.15) $4 \div 1{,}000 = \underline{0.004}$

3.16) $465.1 \div 100 = \underline{4.651}$

3.17) $7\overline{)34.86}$ $= 4.98$

3.18) $21\overline{)640.5}$ $= 30.5$

3.19) $45\overline{)188.1}$ $= 4.18$

3.20) $.12\overline{)47.64}$ $= 397.$

3.21) $.28\overline{).8876}$ $= 3.17$

3.22) $3.7\overline{)1494.8}$ $= 404$

3.23) $\frac{7}{8} = 8\overline{)7.0}$ $= .875 = 0.9$

3.24) $\frac{7}{25} = 25\overline{)7.00}\ .28 = 0.28$

3.25) $\frac{4}{625} = 625\overline{)4.0000}\ 0.0064 = 0.0064$

3.26) $.2 = \frac{2}{10} = \underline{\underline{\frac{1}{5}}}$ Lowest Terms

3.27) $.75 = \frac{75}{100} = \underline{\underline{\frac{3}{4}}}$ Lowest Terms

3.28) $.4 = \frac{4}{10} = \underline{\underline{\frac{2}{5}}}$ Lowest Terms

3.29) $33.6520 = \underline{\underline{33.65}}$

3.30) $7.632 = \underline{\underline{7.63}}$

3.31) $3.336 = \underline{\underline{3.34}}$

3.32) $12.7895 = \underline{\underline{12.79}}$

3.33) $10.7188 = \underline{\underline{10.72}}$

Chapter IV

4.1) $50\% = \frac{50}{100} = \underline{\underline{\frac{1}{2}}}$

4.2) $10\% = \frac{10}{100} = \underline{\underline{\frac{1}{10}}}$

4.3) $80\% = \frac{80}{100} = \underline{\underline{\frac{4}{5}}}$

4.4) $131\% = \frac{131}{100} = 1\ \frac{31}{100} = \underline{\underline{1.31}}$

4.5) $245\% = \frac{245}{100} = 2\frac{45}{100} = 2\ \frac{9}{20} = \underline{\underline{2.45}}$

4.6) $161\% = \frac{161}{100} = 1\frac{61}{100} = \underline{1.61}$

4.7) $.35 = \underline{35\%}$

4.8) $7.9 = \underline{790\%}$

4.9) $21\% = \underline{.21}$

4.10) $23\% = \underline{.23}$

4.11) 5% of \$60.00 = \$3.00

$$\begin{array}{r} 60 \\ \times .05 \\ \hline 3.00 \end{array} = \$3.00$$

4.12) 20% of 320 days = 64 days

$$\begin{array}{r} 320 \\ \times .20 \\ \hline 64.00 \end{array} = \underline{64 \text{ days}}$$

4.13) 15% of 22 pounds = 33.75 or 34

$$\begin{array}{r} 225 \\ \times .15 \\ \hline 1125 \\ 225 \\ \hline 33.75 \end{array} = 33.75 \text{ pounds}$$

4.14) 110% of 50 grams = 55

$$\begin{array}{r} 50 \\ \times 1.10 \\ \hline 500 \\ 50 \\ \hline 55.00 \end{array} = \underline{55 \text{ grams}}$$

4.15) 50% of 228 years = 114

$$\begin{array}{r} 228 \\ \times .50 \\ \hline 114.00 \end{array} = \underline{114 \text{ years}}$$

Chapter V

5.1) $81:x = 45:10$

$$\frac{81}{x} = \frac{45}{10}$$

$45x = 81 \times 10$ $\qquad x = \frac{810}{45}$ $\qquad x = \underline{18}$

5.2) $x:7 :: 350:10$

$$\frac{x}{7} = \frac{350}{10}$$

$10x = 350 \times 7$ $\qquad x = \frac{2450}{10}$ $\qquad x = \underline{245}$

5.3) $5:500 :: x:8340$

$$\frac{5}{500} = \frac{x}{8340}$$

$500x = 8340 \times 5$ $\qquad x = \frac{41,700}{500}$ $\qquad x = \underline{83.4}$

Chapter VI

6.1) $1.1 + 0.8 + 0.5 + 1.2 + 0.8 + 0.7 + 0.5 = \frac{5.6}{7} = \underline{0.8 \text{ Average}}$

6.2) $6.8 + 6.5 + 6.7 + 7.0 + 6.9 = \frac{33.9}{5} = \underline{6.78 \text{ or } 6.8}$

6.3)
5.4
5.6
5.7
5.8
5.9
6.2

$$\begin{array}{r} 5.7 \\ +5.8 \\ \hline 11.5 \end{array} = \frac{11.5}{2} = \underline{5.75 \text{ pounds median}}$$

6.4)
9:20
9:30
9:35
9:45
10:05
10:10

$$\begin{array}{r} 9:45 \\ +9:35 \\ \hline 18:80 \end{array} = \frac{18:80}{2} = \underline{9:40 \text{ Time of Peak Demand}}$$

Chapter VII

7.1) 2 Inches

7.2) 4 Inches

7.3) 6.2 cm.

7.4) 8.8 cm.

7.5) 9 yd. + 11 yd. + 10 yd. = 30 yds.

7.6) 5 ft. + 6 ft. + 6 ft. = 17 ft.

7.7) 2 mi. + 4 mi. + 5 mi. = 11 mi.

7.8) $A = \frac{B \times h}{2} = \frac{5 \times 6}{2} = \frac{30}{2} =$ 15 sq. ft.

7.9) $A = \frac{B \times h}{2} = \frac{3 \times 4}{2} = \frac{12}{2} =$ 6 sq. yds.

7.10) $A = \frac{B \times h}{2} = \frac{7 \times 2}{2} = \frac{14}{2} =$ 7 sq. in.

7.11) 6 ft. + 6 ft. + 6 ft. + 6 ft. = 24 ft.

7.12) 2 yds. + 4 yds. + 2 yds. + 4 yds. = 12 yds.

7.13) $A = s^2$

$A = (7 \text{ in.})^2$ $A = 49$ sq. in.

7.14) $A = s^2$

$A = (21 \text{ yds.})^2$

$A = 441$ sq. yds.

7.15) $A = L \times W$

$A = 5 \times 2$

$A = 10$ sq. in.

7.16) $A = L \times W$

$A = 10 \times 9$

$A = 90$ sq. ft.

7.17) $C = 2 \pi R$

$C = 2 \times 3.14 \times 7$ ft.

$C = 6.28 \times 7$ ft.

$C = \underline{43.96 \text{ ft.}}$

7.18) $C = \pi D$

$C = 3.14 \times 12$ in.

$C = \underline{37.68 \text{ in.}}$

7.19) $A = \pi R^2$

$A = 3.14 \times 17$ in. $\times 17$ in.

$A = 3.14 \times 289$ in.^{2}

$A = \underline{907.46 \text{ sq. in. or } 907.5 \text{ sq. in.}}$

7.20) $A = \pi \frac{D^2}{4}$

$A = 3.14 \times (12 \text{ ft.} \times 12 \text{ ft.})$

$A = \frac{3.14 \times 144 \text{ sq.ft.}}{4}$

$A = \frac{452.16 \text{ sq. ft.}}{4}$

$A = \underline{113.04 \text{ sq.ft.}}$

7.21) $V = S \times S \times S$

$V = 7$ ft. $\times 7$ ft. $\times 7$ ft.

$V = 49$ sq.ft. $\times 7$ ft.

$V = \underline{343 \text{ cu.ft.}}$

7.22) $V = S \times S \times S$

$V = 2$ yd. $\times 2$ yd. $\times 2$ yd.

$V = 4$ sq.yd. $\times 2$ yd.

$V = \underline{8 \text{ cu.yd.}}$

7.23) $V = L \times W \times H$

$V = 5$ yds. $\times 2$ yds. $\times 3$ yds.

$V = 10$ sq.yds.$^2 \times 3$ yds.

$V = \underline{30 \text{ cu.yds.}}$

7.24) $V = L \times W \times H$

$V = 6$ ft. $\times 4$ ft. $\times 3$ ft.

$V = 24$ sq.ft. $\times 3$ ft.

$V = \underline{72 \text{ cu.ft.}}$

7.25) $V = \frac{1}{2}$ (B x H) x L

$V = \frac{1}{2}$ (7 x 9) x 14

$V = \frac{1}{2}$ (63) x 14

$V = \frac{1}{2} \times 882$

$V = \underline{441 \text{ cu.ft.}}$

V = Area x 3rd. dimension

$V = \frac{63 \text{ sq.ft.}}{2} \times 14$

V = 3.15sq.ft. 14 ft.

V = 441 cu.ft.

7.26) $V = 11 R^2 \times H$

V = 11 5 sq.ft. x 11 ft.

V = 3.14 x 25 x 11

$V = \underline{863.5 \text{ cu.ft.}}$

V = Area x 3rd dimension

$V = 11 R^2$ x 3rd dimension

V = 3.14 x 5 sq.ft. x 3rd. dimension

V = 78.5 sq.ft. x 11 ft.

$V = \underline{863.5 \text{ cu.ft.}}$

Chapter IX

9.1) 1000 cu. ft. = 7,480 gal.

1 cu. ft. = 7.48 gal.

9.2) $3 \text{ cu.ft.} \times \frac{7.48 \text{ gal.}}{\text{cu.ft.}} \times \frac{8.34 \text{ lbs.}}{\text{gal.}} = \underline{187.15 \text{ pounds}}$

9.3) Net Weight $\frac{750 \text{ pounds}}{8.34 \text{ lbs./gal.}} = \underline{89.9 \text{ gallons}}$ or $\underline{90 \text{ gallons}}$

9.4) $\frac{1,500,000 \text{ gal.}}{\text{day}} \times \frac{1 \text{ day}}{86,400 \text{ seconds}} \times \frac{1 \text{ cu.ft.}}{7.48 \text{ gal.}} = \underline{2.32 \text{ CFS}}$

9.5) $\frac{450 \text{ gal.}}{\text{min.}} \times \frac{\text{mon.}}{60 \text{ sec.}} = \underline{7.5 \text{ gps}}$

9.6) 1,800,000 lbs./day

$$0.6 \text{ mg/l FL} = \frac{.6 \text{ lbs. FL}}{1{,}000{,}000 \text{ lbs. flow}}$$

$$\frac{.6 \text{ lbs. FL}}{1{,}000{,}000} \times \frac{1{,}800{,}000 \text{ lbs. flow}}{\text{day}}$$

$$\frac{.6 \text{ lbs. FL}}{\cancel{1{,}000{,}000}\ 1 \text{ lbs. flow}} \times \frac{1.8\ \cancel{1{,}800{,}000}\ \text{lbs. flow}}{\text{day}}$$

$$.6 \text{ lbs. FL.} \times \frac{1.8}{\text{Day}} = \underline{\underline{1.08 \text{ lbs/day}}}$$

9.7) $2.5 \text{ mg/l } CL_2 = \frac{2.5 \text{ lbs. } CL_2}{1{,}000{,}000}$

$$\frac{2.5 \text{ mg/l}}{1{,}000{,}000 \text{ lbs. flow}} \times \frac{8{,}300{,}000 \text{ lbs. flow}}{\text{day}}$$

$$\frac{2.5 \text{ mg/l}}{1{,}000{,}000 \text{ lbs. flow}} \times \frac{8.3\ 8{,}300{,}000 \text{ lbs. flow}}{\text{day}}$$

$$2.5 \text{ lbs} \times \frac{8.3}{\text{day}} = \frac{20.75 \text{ lbs. } CL_2}{\text{day}} \text{ or } \frac{20.8 \text{ lbs. } CL_2}{\text{day}}$$

9.8) Grains Per Gallon (G.P.G.) $= \frac{\text{lbs. chemical used}}{\text{flow (MGD)} \times 143}$

$$G.P.G. = \frac{1400}{7.6 \times 143}$$

$$G.P.G. = \frac{1400}{1087}$$

$$G.P.G. = \underline{\underline{1.29}}$$

9.9) Grains Per Gallons $= \frac{\text{lbs. of chemical used}}{\text{flow (mgd)} \times 143}$

$$5 = \frac{x}{0.02 \text{ MGD} \times 143}$$

$$x = 5 \times 0.02 \times 143$$

$$x = \underline{\underline{14.3 \text{ lbs. of chemical}}}$$

SECTION II

PROCESS CALCULATIONS

CHAPTER I

1.1) lbs/day Cl_2 = 0.4 mg/l x 5,300,000 mpd x 8.34 lbs/gal

= 0.4 mg/l x 5.3 MGD x 8.34 lbs/gal

= 17.68 lbs/day CL_2

1.2) lbs/day CL_2 = 0.2 mg/l x 1.0 MGD x 8.34 lbs/day

= 1.67 lbs CL_2/day

1.3) Chlorine Demand = Chlorine Dose - Chlorine Residual

Demand = 1.6 - 0.4 = 1.2 mg/l

1.4) Demand = 0.3 - 0.2 = 0.1 mg/l

1.5) Demand = 1.0 - 0.4 = 0.6 mg/l

1.6) Demand = Dose - Residual

2.6 mg/l = 10 mg/l - Residual

Residual = 10 mg/l - 2.6 mg/l

= 7.4 mg/l

1.7) Residual = Dose - Demand

= 1.2 mg/l - 0.4 mg/l

Residual = 0.8 mg/l

1.8) Given: Residual = 0.2 mg/l
Dose = 1.2 mg/l
Flow = 0.45 MGD
Strength = 68% available chlorine

Find: lbs/day hypochlorite needed

Step 1: lbs/day CL_2 req'd = 1.2 mg/l x .45 MGD x 8.34 lbs/gal

= 4.50 lbs/day CL_2

Step 2: at 68% strength $\frac{4.5}{.68}$ = 6.6 lbs. hypochlorite req'd/day

CHAPTER II

2.1) Given: Dose = 0.8 mg/l
Flow = 2.6 MGD

Find : lbs/day of soda ash

lbs/day ash = 0.8 mg/l x 2.6 MGD x 8.34 lbs/gal

= 17.35 lbs/day soda ash

2.2) Given: Dose = 1.4 mg/l sodium aluminate
Flow = 3.6 MGD

Find : lbs/day Sodium Aluminate

lbs/day sodium aluminate = 1.4 mg/l x 3.6 MGD x 8.34 lbs/gal

= 42.03 lbs/day sodium aluminate

2.3) Given : lbs chemical = 10
gals. water = 5

Find : % strength of solution

Step 1: 5 gallons water x 8.34 lbs/gal = 41.7 lbs water

Step 2: $\frac{10 \text{ lbs. chemical}}{41.7 \text{ lbs. water} + 10 \text{ lbs. chemical}} \times 100 = 19.3\%$

2.4) Given: Flow = 15.2 MGD
Dose = 2 ppm alum

Find : lbs alum per 30 days

Step 1: lbs/day alum = 2 mg/l x 15.2 MGD x 8.34 lbs/gal

= 253.5 lbs alum/day

Step 2: 253.5 lbs/day x 30 days = 7606 lbs per 30 days

2.5) Given: Flow = 6.5 MGD
Tank Dimensions = 60 feet x 20 feet x 15 feet
Tank shape = Rectangular

Find: Detention Time

Step 1: Tank Volume = area (sq. ft.) x depth (ft)
= 60 x 20 x 15
= 18000 cu.ft.

Step 2:

$$\text{Detention Time (hours)} = \frac{\text{tank volume (cu.ft.)} \times 7.48 \text{ gal/cu.ft.} \times 24 \text{ hr/day}}{\text{Flow Rate (gal/day)}}$$

$$\text{Detention Time} = \frac{18000 \text{ cu.ft.} \times \frac{7.48 \text{ gal}}{\text{cu.ft.}} \times \frac{24 \text{ hr.}}{\text{day}}}{6{,}500{,}000 \text{ gal/day}}$$

$$= \underline{.50 \text{ hours}} \text{ (or 30 minutes)}$$

2.6) Given: Flow = .75 MGD
Shape = Circular
Size = 20 ft radius
10 ft depth

Find: Detention Time

Step 1: Tank Volume = area (sq.ft.) x depth (ft.)

$= \pi r^2 \times H$

$= (3.14)(20)(20)(10)$

$= 12560$ cu.ft.

Step 2:

$$\text{Detention Time (hours)} = \frac{\text{Tank Volume (cu.ft.)} \times 7.48 \text{ gal/ft}^3 \times 24 \text{ hr/day}}{\text{Flow Rate (gal/day)}}$$

$$= \frac{12560 \text{ cu.ft.} \times 7.48 \text{ gal/cu.ft.} \times 24 \text{ hr/day}}{750{,}000 \text{ gal/day}}$$

$$= \underline{3.0 \text{ hours}}$$

CHAPTER III

3.1) Given: No. of Filters = 3
Size = 10 ft x 7 ft
Operating = 1 filter out or service
Rate = 280 GPM per filter

Find: gal/ft^2/min. per filter

Step 1: Find area of filters

Area = 10 ft x 7 ft = 70 sq.ft.

Step 2: Find rate

$$\frac{280 \text{ gal/min}}{70 \text{ sq.ft.}} = \underline{4.0 \text{ gal/sq.ft./min. per filter}}$$

3.2) Given: Filter Box = 35 ft. x 25 ft.
Water Drops = 4.5 in. per min.

Find : Rate of Filtration in MGD

Step 1: 35 ft. x 25 ft. = 875 sq.ft. = Area

Step 2: Convert drop into feet:

$$\frac{4.5 \text{ in/min.}}{12 \text{ in/ft.}} = .375 \text{ ft/min.}$$

Step 3: 875 sq. ft. x .375 ft/min = 328 cu. ft./min.

Step 4: $\frac{328 \text{ cu.ft.}}{\text{min.}} \times \frac{7.48 \text{ gallons}}{\text{cu. ft.}} = 2453 \text{ gallons/min.}$

Step 5: $\frac{2453 \text{ gallons}}{\text{min}} \times \frac{1440 \text{ min}}{\text{day}} = 3{,}532{,}320 \text{ gallons/day}$

Step 6: $\frac{3{,}532{,}320}{1{,}000{,}000} = \underline{\underline{3.5 \text{ MGD}}}$

3.3) Given: Filter dimensions Area = 300 sq. ft.
Backwash rate = 15 gallons/sq.ft./min.
Backwash time = 8 minutes

Find : Amount of water for each backwash

Step 1: Find amount of water/min.

$$= 300 \text{ sq. ft.} \times \frac{15 \text{ gallons}}{\text{sq. ft./min}} = 4500 \text{ gallons/min.}$$

Step 2: Gallons per 8 min.

$$= 4500 \frac{\text{gallons}}{\text{min.}} \times 8 \text{ min.} = \underline{\underline{36{,}000 \text{ gallons used}}}$$

3.4) Given: Filter Dimensions
Length = 28 ft.
Width = 21 ft.
Depth = 46 in.
Flow = 2500 GPM

Find: Rate of Rise in Filter

Step 1: Find area = 28 ft. x 21 ft. = 588 sq. ft.

Step 2: Find rate = $\frac{2500 \text{ GPM}}{588 \text{ sq.ft.}} = 4.25$ gallons/sq.ft./min.

Step 3: Convert gallons to cu.ft. = $\frac{4.25 \text{ gallons/sq.ft./min}}{7.48 \text{ gallons/cu.ft.}}$

= .57 cu.ft./sq.ft./min

= .57 ft/min

CHAPTER IV

4.1) Given: Distance = 1500 ft.
Time = 4 minutes

Find : Velocity in ft/min

Step 1: $\text{Velocity} = \frac{1500 \text{ ft}}{4 \text{ min.}} = 375 \text{ ft/min}$

Step 2: Convert minutes to seconds

$$\frac{375 \text{ ft.}}{\text{min.}} \times \frac{1 \text{ min.}}{60 \text{ sec.}} = \underline{\underline{6.25 \text{ ft/sec.}}}$$

4.2) Given: Flow = 350 GPM
Pipe Size = 6 inches

Find : Velocity (ft/sec) $\text{Velocity} = \frac{\text{Distance}}{\text{Time}}$

Step 1: Convert gallons to cu.ft.

$$\frac{350 \text{ gallons/min}}{7.48 \text{ gal/cu.ft.}} = 46.8 \text{ cu.ft./min.}$$

Step 2: Find cross-sectional area of pipe

Area of circle $= \pi r^2$
$= 3.14 \text{ (3in) (3in)}$
$= 28.26 \text{ sq. in.}$

Step 3: Convert square inches to square feet

$$\frac{28.26 \text{ sq.in.}}{144 \text{ sq. in/sq.ft.}} = .20 \text{ sq.ft.}$$

Step 4: Find ft/min

$$= \frac{46.8 \text{ cu.ft./min.}}{.20 \text{ sq.ft.}} = \underline{\underline{234 \text{ ft/min}}}$$

Step 5: Convert minutes to seconds

$$234 \text{ ft/min} \times \frac{1 \text{ min}}{60 \text{ sec.}} = \underline{\underline{3.9 \text{ ft/second}}}$$

4.3) Given: Flow = 600 GPM
Size = 24 inch pipe

Find: Velocity (ft/sec)

Step 1: Convert gallons to cu.ft.

$$\frac{600 \text{ gal/min}}{7.48 \text{ gal/cu.ft.}} = 80.2 \text{ cu.ft./min.}$$

Step 2: Find cross-sectional area of pipe

Area of circle = πr^2
= (3.14) (12) (12)
= 452.2 sq. in.

Step 3: Convert sq. in. to sq. ft.

$\frac{452.2 \text{ sq. in.}}{144 \text{ sq.in./sq.ft.}}$ = 3.14 sq. ft.

Step 4: Find ft/min

$\frac{80.2 \text{ cu.ft./min}}{3.14 \text{ sq.ft.}}$ = 25.5 ft/min

Step 5: Convert minutes to seconds

25.5 ft/min x $\frac{1 \text{ min}}{60 \text{ sec.}}$ = .42 ft/sec.

4.4) Given: Distance = 15,000 ft.
Time = 7 minutes

Step 1: Velocity = 15,000 ft ÷ 7 min. = 2143 ft/min

Step 2: Convert minutes to seconds

2143 ft/min x $\frac{1 \text{ min}}{60 \text{ sec.}}$ = 35.7 ft/second

4.5) Given: Shape = circular
Depth of water = 15 feet
Diameter = 30 feet

Find: Gallons of water

Volume = $\pi r^2 \times h$
Volume (gallon) = (3.14) (15ft) (15ft) (15ft) x $\frac{7.48 \text{ gallons}}{\text{cu. ft.}}$
= 79,269 gallons

4.6) Given: Shape = Cylindrical Tank
Size = 80 ft. height
20 ft. diameter
Paint = 1 gallon cover 150 sq.ft.

Find: Gallons of paint needed

Step 1: Find area of top and bottom

Area = πr^2
= 3.14 (10) (10)
= 314 sq.ft. of each
314 + 314 = 628 sq.ft. (top and bottom)
2 (628) = 1256 sq.ft. to paint inside and outside

Step 2: Find area of side wall
(hint - side wall can be considered a rectangle 80 ft. by the circumference)

circumference = $\overline{2\pi}r$ = (2) (3.14) (10) = 62.8 feet

Area of side wall = 80 ft. x 62.8 ft. = 5024 sq.ft.
Double for inside and outside
Area to paint = (2) (5024) = 10048 sq. ft.

Step 3: Add all area together

10048 sq.ft. + 1256 sq.ft. = 11304 sq. ft. to paint

Step 4: Paint Required

$\frac{11304 \text{ sq.ft.}}{150 \text{ gal/sq.ft.}}$ = 75.36 or 76 gallons of paint

4.7) Given: Volume = 125,000 gallons
Available CL_2 = 70%
Dosage = 50 ppm CL_2

Find: lbs. hypochlorite

Step 1: Find total lbs. of CL_2 required

lbs. CL = Dose (mg/l) x volume
= 50 mg/l x .125 MG x 8.34 lbs/gal
= 52.1 lbs CL_2

Step 2: Find lbs hypochlorite at 70% strength

$\frac{52.1 \text{ lbs CL}}{.70}$ = <u>74.5 lbs hypochlorite</u>

4.8) Given: Length = 4500 ft.
Pipe size = 6 inches
Dosage = 60 ppm CL_2

Find: lbs. of HTH at 70% strength

Step 1: Find volume of water to treat 3 in = .25 ft.

Volume = $\pi r^2 h$
= (3.14) (0.25) (0.25) (4500ft) x 7.48 gal/ft^3
= 6605.7 = 6606

Step 2: Find lbs. CL_2 required

lbs. CL_2 = Dose mg/l x Volume (MG) x 8.34 lbs/gal
= 60 mg/l x 0.0066 MG x 8.34 lbs/gal
= 3.3 lbs CL_2

Step 3: Convert to hypochlorite at 70%

$\frac{3.3 \text{ lbs } CL_2}{.70}$ = <u>4.71 lbs.</u> hypochlorite

CHAPTER V

5.1) 1 psi = 2.3 ft. of head

$$\frac{50 \text{ ft. head}}{2.3 \text{ ft. head/1psi}} = 21.7 \text{ psi}$$

5.2) $F = 31.2 \times H^2 \times L$

$F = 31.2 \times (10)^2 \times 4$

$= \underline{\underline{12480 \text{ lbs}}}$

5.3) Given: Flow = 5.4 MGD
Head = 23 feet
Pump eff. = 90%
Motor eff. = 88%

$$HP = \frac{\text{Head (ft)} \times \text{flow (GPM)}}{3960 \times \text{eff.p.} \times \text{eff.m.}}$$

Find: HP

$$HP = \frac{(23\text{ft}) \times (5{,}400{,}000 \text{ gal/day}) \times \frac{1 \text{ day}}{1440 \text{ min.}}}{3960 \times .90 \times .88}$$

HP = 27.5 HP

5.4) Given: Flow = 500 gpm
Head = 100 ft.
Pump eff. = 70%

$$\text{Brake, HP} = \frac{\text{Flow, gpm} \times \text{H, ft.}}{3960 \times E_p}$$

$$= \frac{500 \times 100}{3960 \times 0.70}$$

$$= \underline{\underline{18 \text{ HP}}}$$

F. BIBLIOGRAPHY

Basic Mathematics and Wastewater Process Calculations, New York State Department of Environmental Conservation, Albany, 1975.

Basic Mathematics, Books I-II, 2nd. ed., J.A. Smith, Encyclopedia Britannica Educational Corporation, Chicago, 1974.

Correspondence Course Manual for Water Plant Operators, 2nd. ed., J.H. Austin, 2nd. ed., South Carolina Water and Pollution Association, Clemson University, Clemson, 1968.

Fundamental Mathematics for Water and Wastewater Treatment Plant Operators, Vol. I, J. Krikpatrick, Ann Arbon Science Publishers, Inc., Ann Arbor, 1975.

Idaho Waterworks Operators Certification Board Training Manual, Professor A.T. Wallace, University of Idaho, Boise.

Programmed Learning Package for Water and Wastewater Treatment Plant Operators, South Dakota Department of Environmental Protection, Pierre, 1977.

Wastewater Treatment Plant Operators Course, Educational Pamphlet No. 4, New England Regional Wastewater Institute, South Portland, 1969.